THE EFFECT OF CRITICAL DISTANCE ON STABILITY CONDITIONS FOR A CRACK AT THE INTERFACE BETWEEN TWO POLYMER MATERIALS

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The aim of this work is to study the behaviour of a crack with its tip at the interface between two polymer materials. A numerical model of a cracked bi-material tension specimen is investigated and different stability criteria are tested. The stability criterion of a general stress concentrator usually needs a relation between the critical value of the generalized stress intensity factor (H_{IC}) and critical value of the stress intensity factor K_{IC} (fracture toughness). This relation is a function of the elastic mismatch of particular materials; the fracture toughness of the main material and the critical distance d ahead of the stress concentrator, where the criterion is applied. Estimation of distance d is usually not straightforward and different authors use different approaches for its determination. Therefore the main aim of this study is the mutual comparison of published approaches for d estimation and to quantify the influence of d choice on the critical load value. The results obtained can lead to a better residual lifetime prediction and safer design of layered structures.

Keywords: *stability criterion, critical distance, bi-material interface, generalized stress intensity factor*

1. Introduction

The use of advanced materials such as composite materials or layered materials has been rising steeply in recent times. The appropriate selection of the material properties of a layered structure can lead to a better mechanical, thermal or chemical resistance in the whole system. The existence of the step change of material properties influences stress distribution near the crack tip and also influences crack propagation through the interface. This work is focused on stability conditions of the crack perpendicular and terminating at the interface between two polymer materials (see Fig. 1) and discusses the estimation of the critical distance d in particular stability criteria.

Under the assumptions of linear elastic fracture mechanics the stress field near the crack tip in a homogenous material can be described by stress intensity factor K [1]. In the case of a perpendicular crack touching the interface a classical approach based on the stress intensity factor cannot be used. Due to a mismatch of the elastic properties of individual material components the value of stress singularity exponent p differs from 0.5 [2–4]. The

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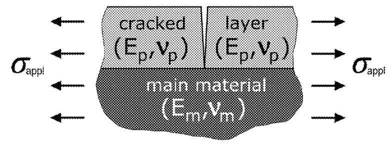

Fig.1: Perpendicular crack touching the interface between two elastic materials

stress distribution around the crack with its tip at the interface between two materials can be expressed as follows [5, 6] :

$$\sigma_{ij} = \frac{H}{\sqrt{2\pi}\, r^p}\, f_{ij}(\theta, p, \alpha, \beta) \ , \tag{1}$$

where r, θ are polar coordinates with the origin at the crack tip, p is the stress singularity exponent, $f_{ij}(\theta, p, \alpha, \beta)$ is a known function, H is the generalized stress intensity factor [MPa m^p], α, β are Dundurs parameters for plane strain conditions, and for a crack with a tip perpendicular to the material interface can be expressed as [7] :

$$\alpha = \frac{\dfrac{E_\mathrm{p}}{E_\mathrm{m}}\,\dfrac{1+\nu_\mathrm{p}}{1+\nu_\mathrm{m}} - 1}{2\,(1-\nu_\mathrm{p})} \qquad \text{and} \qquad \beta = \frac{E_\mathrm{p}}{E_\mathrm{m}}\,\frac{1-\nu_\mathrm{m}^2}{1-\nu_\mathrm{p}^2} \ . \tag{2}$$

The value of the generalized stress intensity factor has to be estimated from the numerical solution of a loaded body with material interface. Its value is determined by comparison of a semi-analytical solution (eq. 1) with numerically estimated values of the corresponding stress component (the crack opening stress component ahead of the crack tip is usually used) [8]. This method is called the direct method and is based on extrapolation of the normalised opening stresses H_I^* to the crack tip H_I, see Fig. 2.

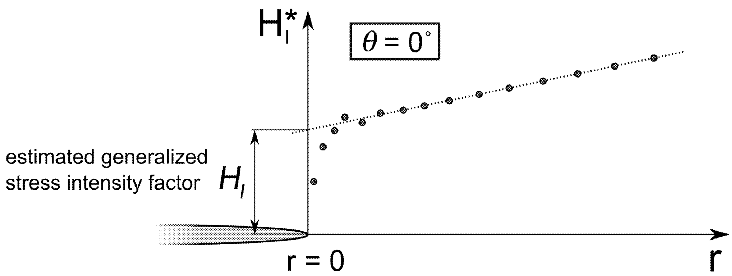

Fig.2: Methodology of the direct method

For determination of the crack behaviour (whether the crack stays arrested at the interface or passes through the interface) the stability criterion should be used. The basic definition of the stability criterion of a brittle homogenous material can be written as follows :

$$K_\mathrm{I}(\sigma_\mathrm{crit}^\mathrm{hom}) = K_\mathrm{IC} \ , \tag{3}$$

where $K_\mathrm{I}(\sigma_\mathrm{crit}^\mathrm{hom})$ is the value of stress intensity factor for critical loading stress $\sigma_\mathrm{crit}^\mathrm{hom}$ and K_IC is the fracture toughness. Critical loading stress is represented by the level of externally applied stress when the existing crack (defect) starts to propagate. For a crack with its tip at the material interface the stability condition (3) can be rewritten to a generalized form as :

$$H_\mathrm{I}(\sigma_\mathrm{crit}) = H_\mathrm{IC}(K_\mathrm{IC}) \ . \tag{4}$$

After that it is necessary to find the critical value of generalized stress intensity factor $H_{IC}(K_{IC})$ as a function of fracture toughness K_{IC} of the material to which the crack will propagate. To find H_{IC} value different approaches were developed in the past.

The first criterion mentioned here is based on Sih's concept of strain energy density (SED) factor. The failure of the material occurs when the value of the strain energy density factor exceeds its critical value; for details see [9]. The critical value of the generalized stress intensity factor derived from this criterion for a crack with its tip at the interface can be expressed as [10] :

$$H_{IC} = \left(\frac{1 - 2\,\nu_m}{(1-p)^2\,(4\,(1-2\,\nu_m) + (g_r - p)^2)} \right)^{\frac{1}{2}} d^{p-\frac{1}{2}}\,K_{IC} \ , \tag{5}$$

where g_r is a known function of elastic material properties [10] :

$$g_r = \lambda - \cos\lambda\pi - \frac{\beta\,[\alpha + 2\,\lambda - (1 + 2\,\alpha + 4\,\alpha\,\lambda^2)\,\cos\lambda\pi + (1+\alpha)\,\cos 2\lambda\pi]}{1 + 2\,\alpha + 2\,\alpha^2 - 2\,(\alpha + \alpha^2)\,\cos\lambda\pi - 4\,\alpha^2\,\lambda^2} \ , \tag{6}$$

$$\lambda = 1 - p \tag{7}$$

and where p is the stress singularity exponent for a crack at a bi-material interface given by the step change of material properties, see [10] for details. The choice of the critical distance d is discussed below.

The second criterion presented here is based on an average opening stress ahead of the crack tip. This criterion assumes that crack behaviour is controlled by the value of the opening stress ahead of the crack tip. If it exceeds its critical value related to the average stress (AS), calculated across the distance d ahead of the crack tip, failure occurs, see [11] for details. The critical value of the generalized stress intensity factor derived from this criterion can be expressed as [11] :

$$H_{IC} = K_{IC}\,\frac{2\,d^{p-1/2}}{2 - p - g_r} \ . \tag{8}$$

Again, the value H_{IC} depends on the fracture toughness K_{IC} of the main material, elastic material constants, the stress singularity exponent p, and on the critical distance d ahead of the crack tip.

The next criterion for determination of crack stability is based on crack mouth opening displacement (CMOD). This criterion is applicable only in the case of a thin protective layer being applied on a thick substrate. The crack mouth opening displacement is related to crack tip opening displacement in this case and can be used as a controlling variable of the crack propagation. The critical value of the crack mouth opening displacement can be expressed as [12] :

$$CMOD_{CRIT}(K_{IC}) = \frac{4\,(1-\nu_m^2)}{E_m}\,K_{IC}\,\sqrt{\frac{2\,a}{\pi}} \ , \tag{9}$$

where a is the crack length (thickness of cracked layer) and E_m and ν_m are elastic material constants (Young's modulus and Poisson's ratio) of the non-cracked substrate. This approach is easily applicable and the critical value of the crack mouth opening displacement can be directly calculated and experimentally measured. This criterion is used for comparison with two previous criteria due to its independence of critical distance value.

2. Critical distance definition

Some stability criteria of general singular stress concentrators use the physical length of the critical distance related to a typical microstructure unit such as the grain size or inclusion spacing. In other approaches authors usually consider critical length d as a size of process or plastic zone ahead of the crack tip. The stability approaches for a crack touching the interface of bi-material bodies are particularly similar to approaches formulated for notched bodies (both cases represent singular stress concentrators). A brief, introductory overview of most common definitions of critical distance is provided below.

The authors of [13] compared the evaluated critical loads obtained by four different criteria with experimental data for a notched specimen made from PMMA (polymethyl-metacrylate) and loaded by biaxial loading. They used the following relations for critical distance estimation:

– the first relation in [13] was derived from the strain energy release rate criterion as:

$$d_1 = \frac{1}{\pi}\left(\frac{K_{\mathrm{IC}}}{1.122\,\sigma_{\mathrm{c}}}\right)^2 , \tag{10}$$

– the second estimation is based on assumptions of maximum energy release rate criterion [13]:

$$d_2 = 0.474\left(\frac{K_{\mathrm{IC}}}{\sigma_{\mathrm{c}}}\right)^2 , \tag{11}$$

– the next approach for the estimation of the critical distance is used in strain energy density criterion proposed by Sih [9, 13]:

$$d_3 = \frac{1-\nu_{\mathrm{m}}}{\pi}\left(\frac{K_{\mathrm{IC}}}{\sigma_{\mathrm{c}}}\right)^2 , \tag{12}$$

– the following estimation of d value is used for prediction of brittle fracture by the criterion developed by Griffith-Irwin [13, 15, 16]:

$$d_4 = \frac{1}{2\pi}\left(\frac{K_{\mathrm{IC}}}{\sigma_{\mathrm{c}}}\right)^2 , \tag{13}$$

– the next estimation of d value was derived for the crack in the case of brittle fracture, see [13, 14] for details:

$$d_5 = \frac{2}{\pi}\left(\frac{K_{\mathrm{IC}}}{\sigma_{\mathrm{c}}}\right)^2 . \tag{14}$$

The same length parameter as eq. 13 is mentioned in the works of David Taylor [17–19], where the author describes his own theory, which he terms 'Theory of Critical Distance' (TCD). A comparison of predicted values of critical loading with experimental data obtained on polycarbonate and steel at low temperature was made in [19] using Taylor's TCD theory.

The critical distance used for prediction of brittle fracture by means of the Griffith-Irwin stability criterion for plane strain condition has the following form [15, 16]:

$$d_6 = \frac{1}{6\pi}\left(\frac{K_{\mathrm{IC}}}{\sigma_{\mathrm{c}}}\right)^2 . \tag{15}$$

The authors of [20] compared estimated critical values for failure of V-notched specimens with experimental data obtained for PMMA and ceramic composite. The criterion based on the strain energy density (SED) in the form derived for plane strain condition was used in the work.

The critical length applied in SED criterion is represented by integration radius [20]:

$$d_7 = \frac{(1+\nu_\mathrm{m})\,(5-8\,\nu_\mathrm{m})}{4\pi} \left(\frac{K_\mathrm{IC}}{\sigma_\mathrm{c}}\right)^2 . \tag{16}$$

The meaning of parameters mentioned in eqs. 10–16 is as follows: ν_m – Poisson's ratio of the main material, K_IC – fracture toughness of the main material, σ_c – tensile strength of the main material.

From a comparison of eqs. 10–16 a general expression for the critical distance can be obtained in the following form (Poisson's ratio typical for polymers $\nu_\mathrm{m} = 0.35$ is considered):

$$d = C \left(\frac{K_\mathrm{IC}}{\sigma_\mathrm{c}}\right)^2 . \tag{17}$$

The values of constant C are summarized in table 1 for individual above mentioned approaches.

	Plane stress					Plane strain	
	eq. 10	eq. 11	eq. 12	eq. 13	eq. 14	eq. 15	eq. 16
C	0.253	0.474	0.207	0.159	0.637	0.053	0.236

Tab.1: A summarised values of constant C determined from eqs. 10–16 ($\nu_\mathrm{m} = 0.35$)

3. Numerical model

To test the stability criteria referred to and critical length definitions a numerical model of a bi-material body was developed. The schema of modelled bimaterial body with perpendicular crack touching the interface between two materials is shown in Fig. 3. The length of the body was chosen as $L = 100\,\mathrm{mm}$, the thickness of the main material was $t_\mathrm{m} = 10\,\mathrm{mm}$ and the thickness of the cracked layer was considered as $t_\mathrm{p} = 2\,\mathrm{mm}$. The Young's modulus of the main material was $E_\mathrm{m} = 800\,\mathrm{MPa}$ and the ratio $E_\mathrm{p}/E_\mathrm{m}$ varied from 0.5 to 2.5. Corresponding Poisson's ratio was considered as $\nu_\mathrm{m} = \nu_\mathrm{p} = 0.35$. The numerical model was loaded by $\sigma_\mathrm{appl} = 1\,\mathrm{MPa}$ and the model assumed ideal adhesion between layers and the step change of material properties across the interface. The model included finite elements strongly non-homogenously distributed in the structure because of the mesh refinement around the crack tip, see Fig. 3. For the calculation plane strain conditions were used. The generalized stress intensity factor values were estimated by the so-called direct method. A sensitivity analysis focusing on the influence of finite element mesh density on the obtained results was performed. Further increase of the mesh density did not bring better accuracy of the results obtained. Only one half of the structure was modelled due to its symmetry.

The critical value of tensile applied stress σ_crit was calculated for each stability criterion and selected critical distance definitions by the following equation:

$$\sigma_\mathrm{crit} = \frac{H_\mathrm{IC}}{H_\mathrm{I}(\sigma_\mathrm{appl})}\,\sigma_\mathrm{appl} . \tag{18}$$

Material properties used in the study corresponded to the high density polyethylene HDPE ($K_\mathrm{IC} = 1.5\,\mathrm{MPa\,m^{1/2}}$, tensile strength $\sigma_\mathrm{c} = 30\,\mathrm{MPa}$).

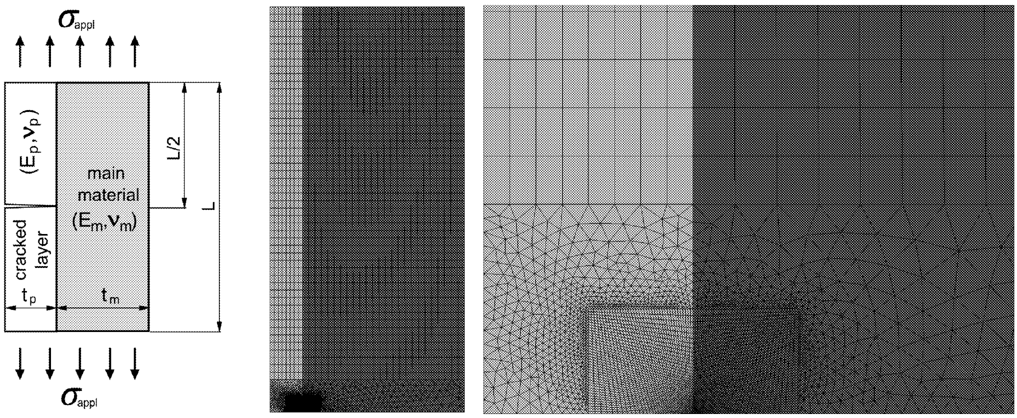

Fig.3: Schema of bi-material body and numerical model with detail of FE mesh refinement around the crack tip

4. Results and discussion

Values for the critical applied stress necessary for crack propagation through the interface between the cracked layer and the main material were estimated. These values depend mainly on the material properties of each layer, on the fracture toughness of the main material and on the critical distance d. Critical stress values calculated by stability criteria based on the strain energy density factor and the average stress ahead of the crack tip were estimated for three different critical distances d. The definitions of d which were used correspond to eq. 10, eq. 13 and eq. 14. Generally, as also seen in Fig. 4, critical stress σ_{crit} increases with a decrease of ratio $E_{\mathrm{p}}/E_{\mathrm{m}}$. This means that the crack can stay arrested at the interface when the material of the protective layer is softer than the material of the substrate. The value of parameter d influences the critical stress value (calculated for Young's moduli ratio from 0.5 to 2.5) only weakly. This result is valid for both stability criteria considered in the study. The maximal difference in resultant critical stress values calculated for a polymer bi- material body was about 18 %.

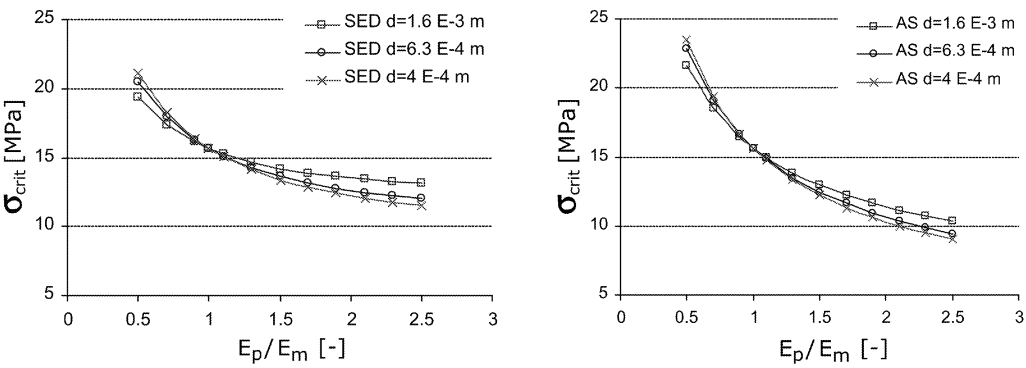

Fig.4: Resultant values of σ_{crit} with regard to ratio $E_{\mathrm{p}}/E_{\mathrm{m}}$ and selected distances d; the critical stress was determined by different stability criteria : by a criterion based on strain energy density factor SED (left figure) and by a criterion based on the average stress ahead of the crack tip AS (right figure)

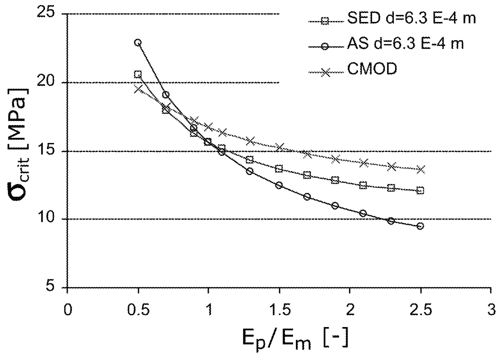

Fig.5: Resultant values of critical stress $\sigma_{\rm crit}$ obtained by application of different stability criteria for given critical distance ($d = 6.3\times10^{-4}$ m)

Resultant values of $\sigma_{\rm crit}$ for both applied stability criteria are shown in Fig. 5. The choice of d distance corresponded to eq. 10. The maximal difference between resultant $\sigma_{\rm crit}$ values was reached for ratio $E_{\rm p}/E_{\rm m} = 2.5$. The critical stress values calculated from mouth crack opening displacement criterion are independent of the critical distance parameter. Calculated data from this criterion show a smaller slope with regard to ratio $E_{\rm p}/E_{\rm m}$ in comparison to other two applied approaches. The criterion based on the strain energy density factor and the criterion based on average stress ahead of the crack tip better corresponds to CMOD criterion for higher value of parameter d. We should point out that the accuracy of the CMOD criterion decreases with the increase of width of the cracked layer. This criterion is suitable mainly in the case of thin coatings applied on the thick material of substrate (e.g. protective coatings).

The effect of the critical distance on $\sigma_{\rm crit}$ obtained using the SED criterion is shown in Fig. 6. It can be seen that the influence of the critical distance parameter d on critical stress increases with an increase of the elastic mismatch of the bi-material body. The influence of the critical distance d is not strong, if we take into account the fact that Fig. 6 represents a parametric study (studying the sensitivity of the given stability criterion on choice of d) with an unrealistic range of parameter d. The range contains several orders of quantity d.

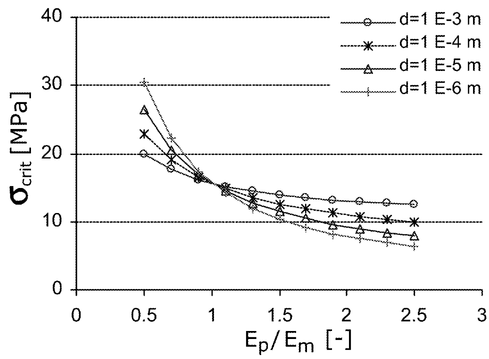

Fig.6: Sensitivity of SED stability criterion on choice of critical distance d (a parametric study)

It is evident from Table 1 that the difference between various approaches of d estimation is not high and all estimated values of d are of the same order. Therefore, only the knowledge of the d order is sufficient for practical application of stability criteria in the case of polymer layered structures, where the usual ratio between Young's moduli $E_{\mathrm{p}}/E_{\mathrm{m}}$ ranges from 0.5 to 2.5 .

5. Conclusions

The stability conditions of a crack penetrating through the interface between two polymer materials were studied and numerically investigated by means of FEM. The problem was approached under the assumptions of (generalized) linear elastic fracture mechanics and the concept of a generalized stress intensity factor was used. For determination of crack behaviour at the material interface (if the crack will stay arrested at the interface or will penetrate through the interface) the stability criteria were used. Critical distance is one of the important parameter, which it is necessary to know for estimation of the critical stress for crack propagation through the interface between two materials. This paper focuses mainly on the influence of the critical distance on the resultant values of critical stress for crack penetration through the interface taking into account different approaches for critical distance estimation available in the literature. The results obtained can lead us to the following points in conclusion:

- the maximal difference of the critical stresses σ_{crit} obtained by the strain energy density factor stability criterion and a criterion based on the average stress ahead of the crack tip for different values of critical distance d obtained from the approaches found in the literature was about 18 % for selected materials (polymers for pipes manufacturing).
- the value of length parameter d taken from the literature and corresponding to the size of the process zone in front of the crack tip has a relatively small influence on the critical stress for crack propagation through the interface of layered polymer material, where the Young's moduli ratio $E_{\mathrm{p}}/E_{\mathrm{m}}$ usually ranges from 0.5 to 2.5 .
- the sensitivity of stability criteria on the choice of d was studied. For selected values of critical distance d from 10^{-3} to 10^{-6} m the resultant values of σ_{crit} were calculated. For determination of σ_{crit} two different stability criteria were applied: criterion based on the strain energy density factor and criterion based on average opening stress value ahead of the crack tip.

Generally, it can be concluded, that the effect of the critical distance on the resulting critical stress is weak and with good accuracy any of the published approaches tested here can be used. The approaches applied correlate the critical distance with a size of the process zone or damage zone in front of the crack tip. Only the knowledge of the critical length order is necessary for satisfying application of stability criteria for a crack touching the interface between two materials. The results of this study can lead to a better estimation of crack behaviour in the case of crack propagation near/through the interface between two polymer materials and finally to the better material design of layered structures with respect to the better resistance against crack propagation.

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