1. EXPERIMENTAL MECHANICS AS A BRANCH OF SCIENCE

1.1. Experimental mechanics and its benefit

If the product has to succeed in the demanding global market, where the supply usually exceeds the demand, it is dependant (besides its price and availability) on its high technical level and quality. In order to achieve this aim it is very necessary to coordinate and also systematically ensure series of processes and activities resulting in products with high-level added value. These processes affect the quality in all stages of the product life cycle (research, development, production, use, reconstruction, liquidation, etc.). The product capabilities, which a designer, constructor, calculating staff, can most significantly influence, are mainly reliability (in the broad sense of the word) involving failure-free operation, safety and service life. According to their knowledge, skills, possibilities and many other factors having impact on their activities, the staff „breathes into" the product a certain rate of inherent reliability. The success of this process is to a considerable extent affected by the quality of applied experimental work in connection with solving mechanical engineering problems that can occur during designing and examination of the product.

Rarely it is possible to meet an objection and misgiving about usefulness of experimental analyses. Expenses, which are related to this activity, are not negligible it is undisputed. First-rate human labor, investment, expendable property, charges, special training of working staff, information – this all cost so much money. There is "a small but", we can recall in this relation known truth: "There is nothing more expensive than uncertain and false information". Well, what we are able to put on the side of benefits? Collectively it can be said, that those are:

- Enhanced quality,
- Longer life-time,
- Enhanced trust of consumers,
- Increased attraction of the article,
- Lower weight of designed construction,
- Fewer problems with legislative (standards and specifications).

The cost value is shown in a balance sheet, which, however, cannot reflect the above-mentioned benefits because it is very difficult to express their value in money. Therefore higher management is sometimes not aware of them and it can make somebody think that the experimental department is not productive.

1.2. Main tasks of experimental mechanics

Here, we will only deal with a problematic related to mechanics of solid. Mentioned will be this range of problems:

Input data achieving (for computational modeling)

One of basis for dimensioning and judging is knowledge about service conditions – first of all quantity of loads time behavior in its running. With the help of topical measuring technology it is able to realize investigation and research not only on static load construction, but also for dynamic loads with high frequencies. Particularly significant it is for construction with stochastic character of its stress-deformation response. Also with the help of personal computers, as an integral part of measuring chain, is this analyze purely self-evident. Global investigation on very complicated structure under operation is neither simple nor cheep matter. The outputs are priceless for design of other same or similar equipment.

The other irreplaceable part of experimental analyses is achieving of needful material data. In this relation is necessary to realize that effect of computational modeling is depended on quality of the input data. Finite elements method (FEM) with its possibilities of more accurately stress and strain analyses, with the solving capacity for more complex and optimization tasks, points out the meaning of experiment. The quality of exp. analyze becomes one of the limited factor for using FEM.

a) Results verification of computational modeling

For a new design of existing construction is also very essential mutual relation between experimental and computational modeling. Usefulness of the construction for giving purpose is able to verify as late as by device-prototype examination, but this procedure is not purposeful for complex and expensive equipment or even for machinery, where pertinent error could threaten health or actually lifetime of people. Well, by experimental modeling can be:

- Verified computational theory and its basement. Here is necessary to connect theory presumption, geometry of the model and loading so that expected character of bindings could be fixed.
- Verified handiness and suitability of the computational model for giving technical performance and realized deviations between solution and experiment. This model is able to compare different construction variants, modification of details, systems of connections etc. In that model is also possible to specify total result of all loads, where do not govern superposition theorem and real construction behaving within an emergency conditions.

Experimental research on the real construction can be then realized for verifying its reliability, for determining of the local loads, stresses concentrations, natural frequencies and modalities, effect of clearance in the bindings, effect of human factor etc. Usually by this way can be found out general result of many influences – inclusive of mounting stresses, self-weight etc.

Exactly so can be refused images, that FEM is able to solve each and every problem. It is necessary to comprehend computational and experimental modeling in its great integral value.

b) Compensation of computation

Experimental analyze is one of the best ways to receive a basis for construction dimensioning, where its computational modeling is unfairly difficult, too long or impossible at all.

c) Monitoring and diagnostics

For exacting and important constructions should be requirement their complex experimental investigation at getting to operation essential matter of course. Also later – in operation – in agreement with construction character and load highness at dangerous places is possible with help of action analyzers observe the process of loading, rise of damage, to signalize overloading and in case of need automatically put aside the whole system. Purposeful construction monitoring then use identification its loading and also dynamics characters. Computational model by this way can be integrated to experiment process.

d) Receiving of the new pieces of knowledge

Finally, experimental methods are very useful instrument in the process of cognition acting operations, for receiving of the new pieces of knowledge and also for discovery the new rules of law, which can be used at future design of mechanical systems. To know experimental mechanics only as results verifier received by other methods or more economical solver of determined tasks would be only narrow view to its main theme.

These approaching problems can be seen in all lifetime parts of every product. They are connecting to all rows of other science levels – quality control, checking of industrial process, material tests, damaging analyze, experts systems and the others.

1.3. Using of experimental mechanics methods

Nowadays, the using area of experimental mechanics is growing up to the other levels of interest, such as:

- Fracture mechanics
- Biomechanics
- Composite materials
- Modal analyze
- Residual stresses
- Experimental analyze
- Diagnostics of a technical products
- Experiment for product quality reservation

Only using the newest approaching and methods, mainly optical methods frequently solve specific conditions and problems.

1.4. Specific outlines of the topical experimental methods

For their limitations is nowadays possible to say:

- The meaning of classical experimental methods goes on,
- Renaissance is not only what now happens, but big development of the optical methods occurred,
- Experiment is:
  - Automatic,
  - PC supported,
  - Interactively controlled,
  - Systematically directed,
- A new requirements rise up for software equipment – mainly for its reliability, freedom, balancing, universality and also specialty.

Constantly growing experimental measuring systems with the using of topical computer technology are taking with another dangerous: over effectively and good looking graphs and also with online computed results can not be seen that for instance measured places were changed or signal was measured in false units.

1.5. Further expected development

Character of other development and research of experimental methods is not simple matter. These days position of that is mainly result of recent theories and they had been results of days before recent needs. Future methods will not be an extrapolation of today’s methods, but their principles are unknown now.

In all views can be seen that penetrative advancement is occurred in optical methods. New stimulation gives also methods such as acoustic emission. Electrical methods are holding their purpose and obviously will get better their sense. On the other hand, the methods founded on magnetic principal have no possibility measuring of the extremely small changes in a magnetic field.

1.6. The errors in experimental modeling and their removing

Significant source of the errors in experimental modeling can be:

- Marginalization of important activities in a tasks of experiment,
- Underestimation of experiment sense in computational modeling and so in solving of engineers' problems,
- Poor connection between theoretical and experimental solution.

It is impossible to neglect other realities as well:

- Experiment must not be self-purposeful (further deeper and deeper specialization, also rising complication of the devices, leads to that). The main aim is not only to know stresses, but correctly to translate it.
- It is needed to have any idea about stresses, which we solve. Without preliminary expected functions of stresses we can easily put the errors to that. For example – in pressure tanks (where is membrane and also bending stress) it is possible to do, that onto outer surface we measure their difference that will be small, on the contrary onto inner surface the sum of these parts can be bigger than permissible value. Another time eventually we can also do a big mistake by neglecting of the stress gradient for a measuring gage with a large platform.
- Repeated errors can also occurred by false modeling of boundary and operating conditions in a research. By neglecting or false translating of the shear friction, by false using of superposition theorem in the system with any clearances or plastic deformation, by false image about mutual force activity in the system of solids, the results can be totally depreciated. Generally, it is very difficult to experimentally modeled single examples of tension.

It is not right to understand it as a mere experimental routine. In order to be a successful worker mean that he has to have a true knowledge about mechanics, elasticity and solidity, he has to know well mechanical properties of used materials in different operation conditions. Using of this knowledge, self-experiences and hard work we are able to minimize rise of blunders for design and realization of the experiment. He should always have had a clear image about questions, which his research should give an answer. It will let him to work more effectively where a big number of measured data is not the right purpose.

1.7. Experimental mechanics in the system of engineers education

For reasons given is undisputable importance of experimental mechanics and also need first-rate training of school-leavers from our high technical schools. Topical state in this direction is possible to assess as not good enough. In a basic level of study is mostly possible to give only brief information about experimental mechanics and its skills. Better situation exists in any specializations, which have this same or similar subject right in contents. Unfortunately, not even here is not in right sense assessed the main task and importance of experimental mechanics. Detachedly said, may be that the best conditions for that is in specialization of applied mechanics at technical universities. At the level of postgraduate study are these conditions good as well and to problematic about experiment is given due care.

Well, requirements for getting this state better should not be untried. To be sure modern specialist at area of experimental mechanics is possible to compare with a doctor – not frequently is visited only for his self-satisfaction, but people come to him when they do not feel themselves really good. In very similar position are specialists in a branch of stress analyze and limiting states. They are call to the construction only when there is dangerous of its breakdown. But it is of course not the right aim to over glued whole construction by strain gage. It is needed clearly assess the whole mechanism and its behavior, correctly judged where is an error and solves the cause of problems. Good doctors and in the same way also good workers for experimental mechanics must start from similar intellectual basement – from the depth of science knowledge self-relations in life organism or in lifeless construction. And also they have to broaden out their special knowledge.

The quality of that training is closely connected to the technical level of school laboratory accessories. A really good help in this direction is close cooperation our institutes with the abroad firms and companies as well.

2. THE SUMMARY OF MORE SIGNIFICANT METHODS IN EXPERIMENTAL MECHANICS

The area of interest is here mainly research of strain, stresses, displacement, bending parameters and also the force activity connected with it, such as forces, pressures, gyroscopic moments etc. The most useable are now mainly electrical methods and also optical methods. To those is here paid the biggest attention. Other methods will be mentioned only briefly.

2.1. Electrical methods

2.1.1. Assessment of the electrical methods

Their preferences are indisputable. So it is – high accuracy, sensitivity and measuring speed even with high number of measured places, possibility of output signal to be analog or digital. This all is indeed connected to higher investment and with higher operation costs of measuring chains and equipment, higher requirements for expertise training of service staff.

2.1.2. Transducers

In nowadays we can meet different transducers and sensors of several generations.

I. Generation transducers are using fundamental principals and phenomena. Their development is really in principle ended up. Not frequently we can come together with a new technology, construction or even with new physical phenomena.

II. Generation transducers are considered semiconductor sensors; their accession has started about year 1950. They have higher sensitivity, accuracy and also miniature sizes. CCD sensors have been used for visible, infrared and ultraviolet area. The development of this type of transducers generation has not been ended up.

III. Generation of transducers is sensors optical-fibred. On the contrary with the others, here the output signal is luminous flux. Using of the light guide for signal transmission has some advantages, which are mainly possibility to transmit the signal on longer distance, bigger transfer bandwidth, problem ablation with electrical and magnetic disturbances. This type of sensors could have much higher sensitivity and much smaller sizes then the others.
In these days, we can come together so-called smart sensors in which the sensitive element is in one compact body with the circuits for analyze, unification and also for signal processing.

Currant using of biological principles to daily praxes leads us to using of biosensors, which are done with biological and physical element combination. It will evidently rise up to IV. Transducer generation. The biggest expansion has had the resistive sensors, from which above all for experimental stress-analyze resistive strain gage. World’s companies that produce resistive strain gages offer wide spectrum of measuring gages satisfying user’s requirements:

- As local stress extreme at the places of their concentration and also stress-gradient (measuring chains). That’s what their platform length is from 0,5mm to 150mm;

- The cross strain gages for two dimensions – for knowing directions of principal stresses, the strain rosette gages for three dimensions – for not knowing directions of the principal stresses, or finally special sensors – for membrane stresses, for measuring in the strong magnetic field, research of residual stresses;
- For common temperatures, low and high temperatures, depending on gage platform material and technology of application. Normal welding strain gages are usable to temperature 250-300^{o}C and special then for 650^{o}C (of course for higher price). Some problems with temperature compensation fall away by using of self-compensating strain gages;
- By incidence of an atomic irradiation.

Very similar to those are temperature sensors, transducers for observation of fatigue damage rising, sensors for gap indication and also for monitoring of its rising.
Possibility of using this metal strain gages is not only for experimental stress analyze but for producing different sensors of forces, pressures, gyroscopic moments etc.
Though semi conductive gages have about two orders higher sensitivity, but for its fragile elements, temperature dependence and higher price have been used only for making of transducers.

Inductivity and capacity types of sensors have been used mainly for measuring of small displacement and for some load cells.
The domain of semiconductor sensors is level of acceleration measuring and also for pressure transducers in extreme conditions etc.

2.1.3. Processing and adjustment of an electrical signal

Digital technique coming to practice markedly influenced the apertures construction for adjustment and processing of an electrical signal. By rising accuracy and speed of A/D converters the analog-digital interface shifts from output of measuring amplifier to closely behind input-amplifier and digital technique picks over the functions of analog circuits like this. The conditional moment was surely miniaturization of computer technique to one-chip's microcomputer, that is possible to mount them to proximity of analog-circuits, where control their function, adjusting and saving their parameters. Microcomputer speed is able to realize major part of basic operation with signals in a real time. In this way is possible to judge the results at once after measure ending.

These topical converters cover by their high resolution the whole measuring range of all sensors without previously needful sensitivity switching of input amplifier. If the output is required in analog form, for disposition are also in the same quality D/A converters. The firms that offer digital measure technique mostly provide the software as well. So, we are able in the interface adjust the parameters of measuring central, to start and save its results on the disk unit in PC in agreeable format. Quite a number of these software include the mathematical as well as graphical instruments for processing and presentation of the results. The firm National Instruments has chosen another approach that can create by using of interface LabView virtual measuring chain by connecting of inputs and outputs blocks performing measure, computational and presentable modulus in the graphical form.

2.2. Optical methods

[Hrabovský, 40. mezinárodní konference „Experimentální analýza napětí", Praha 2002]

2.2.1. Introduction

Interferential and diffractive phenomena of luminous el.-mag. waves has been in optics aged phenomena, that can inspired the long generation of opticians, physicist and technician to new approaching to describe these prodigy.

Classical optics describes interferential and diffractive phenomena with the help of ideal coherent and ideal no coherent light pencil, newly then by semi-coherent light pencil. Superposition of ideal coherent or semi-coherent light pencils lets observe interferential figure. At interferential figure as a rule we require time-stationary and same frequency of light interf. pencil, which can be easily provide by requirement of one primary coherent source of lightning. In example of using incoherent light pencil the interferential image is not visible. But it is possible to observe the activities that result from superposition of these light fields.

Generally, the best division of that optical methods and their technical variant is according to physical principle that also well describe their historical development.

2.2.2. Classic optical method

A period before discovery of phenomenal source of coherent light – laser was characterized by following groups of methods:

a) Optical interferometer
   These are generally divided to:
   - Twin-benched, for example Michelson's, Mach-Zehnder's, Sagnac's etc. From that basic types are further derived different technical variants of interferometers,
   - Many-benched,
   - Heterodynes,
   - Others: to this group can be included special classical interferometers like is Newton's, or devices developed later as a laser anemometer that is called LDA.

b) Stereo-metrical and stereo-foto-grametrical methods those are determined for measuring of deformation, stresses, torsion and so on.

c) Moiré,

d) Photoelasticimetry (transmission, reflective)

Previous methods have been using super positional theorem of interacted waves or polarization. Classical methods were dependent on moderate mono-chromatics source of light. Discovery of the laser light and its technical use in measurement gave a renaissance to those methods, respectively in cooperation with methodical approaching lately discovered methods – as holography, and also use of the new optoelectronic elements, computer technique, mathematics algorithms brought new applications. Following methods are based on the principle of interferometry coherent waves or quasi-coherent optical waves.

2.2.3. Holography and holographic interferometry

Many well-known ways to divide the holography and holographic interferometry are based on two fundamental procedures. They are – Real-time method and Method of twin-exposition. On their principles are based other methods like Time-average method, Stroboscopic method and also Hyperbolic method.

Holography has had many expectations and also many of them have been realized. In cooperation with other classical interferential methods the new holographic methods have brought significant simplification for experimental work and using in mechanics, but this method needs highly qualified service staff.

2.2.4. Coherent-granulity methods

Further development of holographic methods, mainly using of speckle holography and its variants, is very quick due to possibility of using modern technical and optoelectronic elements.

More newly, these methods are often called coherent interf. methods or coherent-granulity methods respective speckle methods. These methods use statistical characters of optical fields of the coherent granulity (speckle). From this point of view those methods are divided to:

- *Speckle interferometry*; this method is based on principle interferometry of coherent fields where the outputs are systems of simple interf. strips with speckle's structure,
- *Statistical correlative speckle method*; the outputs of this method is not characteristically interf. figure, but it is working in two or more fields of coherent-granulity that are statistically evaluated.

2.2.5. Conclusion to the optical methods

Technical progress in the branch of light sources, detectors and computational technology further makes possible also the new view on applications mainly in traditional technical levels.

Topical using of the optical experimental methods in mechanics is commonly evident, where the advantage is surely perspective of variability, contact-less and high level of measured data processing. Expansion of using is possible to expect in a branch of fracture mechanics, biomechanics, and residual stresses, in research of composite materials, quality and so on.

2.3. Brittle paints

Application of the brittle paints is modest and quite simple if they are used for qualitative analyze of stresses. They are suitable for receiving preliminary information about field of stresses before strain gages application. If they are used for quantitative analyze, they have a big number of limitations and therefore they are compensated for difficult task.

2.4. Radiography

Radiography is used besides the structures analyze for stress analyze and mainly for researching of residual stresses. This method is based on diffraction of RTG rays in polycrystalline materials and by this way it is possible to specify distance of atomic planes and also their changes in consequence of elastic deformations. If there are some plastic deformations and their torsion we will not have the possibility to catch anything.

2.5. Acoustic emission

In some cases it is needed in construction or in its part to observe certain activities or states connected with important points of working diagram of construction material (for example yield limit or strength limit) or we need to indicate possibility of errors. Achievement of those states or positions of stresses is not only due to the load quantity but also with combination micro and macro-errors of used material. These errors can be expressed as a soft ultra-sound signal receiving to high ultra-sound spectrum. The signals of acoustic emission are visible if there is any discontinuity in the material. Those causes are:

- Rise and development of micro-defects,
- Shift fazes in the material,
- Rise of creep zones,
- Rise of deformed areas in polymer materials,
- Breaking of fibers in composite materials.

To those then match the energy and frequency of detected signal, from which is possible to analyze the cause and place of process.
Acoustic emission method is used partly for checking of constructions and also for checking of technology.

2.6. Ultra-sound methods

So, we often use these ultra-sound methods for nondestructive materiology. But it is possible to use them for measuring of material thickness and distances, the liquid flow, residual stresses and also for elastic constant of materials.

2.7. Thermal emission

This method is often designated by abbreviation from English title – Stress Pattern Analysis by Thermal Emission as SPATE. The method is based on thermo-dynamics manifestations thermal change by cyclic loading at construction. Signal taken without contact is related to the first invariant of stresses tensor. Sensitivity of this method is 1 MPa for steel and 0,4 MPa for aluminum and its alloys. Besides stress analyze can be SPATE method used in the area of crack spread and for optimization measuring.

2.8. Hybrid methods (experimentally-numerical methods)

This cooperation for solving of the physical problems is very hopeful and also economically acceptable. Using of hybrid methods is promising mainly for nonlinear dynamics problems and tasks, also for fracture mechanics and ductile violation.

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3. EXPRESSION OF UNCERTAINTY AND ERRORS IN MEASUREMENT

3.1. Foreword

In 1978, recognizing the lack of international consensus on the expression of uncertainty in measurement, the world's highest authority in metrology, the International Committee for Weights and Measures (CIPM), requested the International Bureau of Weights and Measures (BIPM) to address the problem in conjunction with the national standards laboratories and to make a recommendation.
To develop a guidance document based upon the recommendation of the BIPM Working group on the Statement of Uncertainties, which provides rules on the expression of measurement uncertainty for use within standardization, calibration, laboratory accreditation and metrology services.

The purpose of such guidance is:

- To promote full information on how uncertainty statements are arrived at,
- To provide a basis for the international comparison of measurement results.

3.2. Introduction

When reporting the result of measurement o a physical quantity, it is obligatory that some indication of the quality of the result be given so that those who use it can assess its reliability. Without such an indication, measurement results cannot be compared, either among themselves or with reference values given in specification or standard. It is therefore necessary that there be a readily implemented, easily understood, and generally accepted procedure for characterizing the quality of a result of measurement, that is, for evaluating and expressing its uncertainty.

The ideal method for evaluating and expressing the uncertainty of the result of a measurement should be

- Universal: the method should be applicable to all kinds of measurements and to all types of input data used in measurements.

The actual quantity used to express uncertainty should be

- Internally consistent: it should be directly derivable from the components that contribute to it, as well as independent of how these components are grouped, or the decomposition of the components into subcomponents.

The quantity should also be

- Transferable: it should be possible to use directly the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used.

3.3. Definition of uncertainty

The word “uncertainty” means “doubt”, and thus in its broadest sense “uncertainty of measurement” means doubt about the exactness or accuracy of the result of a measurement. Because of the lack of different words for this general concept, for example, the standard deviation, it is necessary to use the word “uncertainty” in these two different senses.

The definition of the term "uncertainty (of measurement)" developed for use in this Guide and adopted by the current VIM is the following:

Uncertainty (of measurement)
A parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

3.4. Terms specific to this Guide

In general, terms that are specific to this Guide are defined in the text when first introduced. However, the definitions of six of the most important specific terms are given here for easy reference.

Standard uncertainty
Uncertainty of the results of a measurement expressed as standard deviation.

Type A evaluation (of standard uncertainty)
Method of evaluation of a standard uncertainty by the statistical analysis of a series of observations

Type B evaluation (of standard uncertainty)
Method of evaluation of a standard uncertainty by means other than the statistical analysis of a series of observations.

3.4.1. Combined standard uncertainty

Standard uncertainty of the result of measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities.

3.4.2. Expanded uncertainty

Quantity defining the interval about result of a measurement within which the values that could reasonably be attributed to the measurand may be excepted to lie with a high level of confidence.

3.4.3. Coverage factor

Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty.

3.5. Basic concepts

3.5.1. Measurement

The objective of a measurement is to determine the value of measurand, that is, the value of the specific quantity to be measured. A measurement therefore begins with an appropriate specification of the measurand, the method of measurement and the measurement procedure.

In general, the result of measurement is only an approximation or estimate of the value of the measurand and thus is complete only when accompanied by a statement of the uncertainty of that estimate.

In practice, the specification or definition of the measurand depends on the required accuracy of the measurement. The measurand should be defined with sufficient exactness relative to the required accuracy so that for all practical purposes its value is unique.

In many cases, the result of a measurement is determined on the basis of repeated observations. Variations in repeated observations are assumed to arise from not being able to hold completely constant each influence quantity that can affect the measurement results.

The mathematical model of the measurement procedure that transform the set of repeated observations into the measurement result is of critical importance since, in addition to the observations, it generally includes various influence quantities that are inexactly known. This lack of knowledge contributes to uncertainty of the measurement result along with the variations of the repeated observations and any uncertainty associated with mathematical model itself.

This Guide treats the measurand as a scalar (a single quantity). Extension to a set of related measurands determined in the same measurement procedure requires replacing the scalar measurand and its variance by a vector measurand and covariance matrix. Such a replacement is considered in this Guide only in the examples.

3.5.2. Errors, effects and corrections

In general, a measurement procedure has imperfections that give rise to an error in the measurement result. Traditionally, an error is viewed as having two components, namely, **a random component** and **a systematic component**.

Random error presumably arises from unpredictable or stochastic temporal and spatial variations of influence quantities. The effects of such variations hereafter referred to as random effects, give rise to variations in repeated observations of the measurand. The random error of a measurement result cannot be compensated by correction but increasing the number of observations can usually reduce it, its expectation or expected value is zero.

Systematic error, like random error, cannot be eliminated but it too often be reduced. If a systematic error arises from a recognized effect of an influence quantity on a measurement result, hereafter referred to as a systematic effect, the effect can be quantified and, if significant in size relative to the required accuracy of the measurement, an estimated correction or correction factor can be applied. It is assumed that after correction, the expectation or expected value of the error arising from a systematic effect is zero.

It is assumed that the result of a measurement has been corrected for all recognized significant systematic effects.

3.5.3. Uncertainty

The uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand. The result of a measurement after correction for recognized systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects.

In practice there are many possible sources of uncertainty in a measurement, including:

- Incomplete definition of the measurand;
- Imperfect realization of the definition of the measurand;
- Sampling – the sample measured may not represent the defined measurand;
- Inadequate knowledge of the effects of environmental conditions on the measurement procedure or imperfect measurement of environmental conditions;
- Personal bias in reading analogue instruments;
- Instrument resolution or discrimination threshold;
- Values assigned to measurement standards and reference materials;
- Values of constants and other parameters obtained from external sources and used in the data reduction algorithm;
- Approximations and assumptions incorporated in the measurement method and procedure;
- Variations in repeated observations of the measurand under apparently identical conditions.

These sources are not necessarily independent and some may contribute to the last source. Of course an unrecognized systematic effect cannot be taken into account in the evaluation of the uncertainty of the result of a measurement but contributes to its error.

Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties classifies uncertainty components into two categories based on their method of evaluation, "A" and "B". These categories apply to uncertainty and are not substitutes for the words "random" and "systematic". The uncertainty of a correction for a known systematic effect may be obtained by either a Type A or type B evaluation, as may be the uncertainty characterizing random effect.

The purpose of the Type A and Type B classification is to indicate the two different ways of evaluating uncertainty components and is for convenience of discussion only; it is not meant to indicate that there is any difference in the nature of the components resulting from the two types of evaluation. Both types of evaluation are based on probability distributions and the uncertainty components resulting from each type are quantified by a standard deviation or a variance.

The estimated variance u^2 characterizing an uncertainty component obtained from a Type A evaluation is calculated from a series of repeated observations and is familiar statistically estimated variance s^2. The estimated standard deviation u, the positive square root of u^2, is thus $u = s$ and for convenience is sometimes referred to as a Type A standard uncertainty, For an Uncertainty component obtained from a Type B evaluation, the estimated variance u^2 is evaluated using available knowledge, and the estimated standard deviation u is sometimes referred to as a **Type B standard uncertainty**.

Thus Type A standard uncertainty is obtained from probability density function derived from an observed frequency distribution, while a Type B standard uncertainty is obtained from an assumed probability density function based on the degree of belief that an event will occur. The two approaches are equally valid interpretations of probability.

The total uncertainty of the result of a measurement, termed **combined standard uncertainty** and denoted by u_c, is an estimated standard deviation equal to the positive square root of the total variance obtained by summing all variance and covariance components, however evaluated, using the law of propagation of uncertainty.

To meet the needs of some industrial and commercial applications, as well as requirements in the areas of health and safety, an **expanded uncertainty *U***, whose purpose is to provide an interval about the result of a measurement within which the values that could reasonably be attributed to the measurand may be expected to lie with a high level of confidence, is obtained by multiplying the combined standard uncertainty u_c by a **coverage factor *k***. The choice of the factor k, which is usually in the range 2 to 3, is based on level of confidence desired.

3.6. *Evaluating standard uncertainty*

3.6.1. Modeling the measurement procedure

In most cases a measurand Y is not measurable directly, but depends on N other measurable quantities $X_1,\dots,X_N$ through a functional relationship f:

$$Y = f(X_1,\dots\dots\dots X_N) \tag{1}$$

The input quantities $X_1,\dots,X_N$ upon which the output quantity Y depends may themselves be viewed as measurands and may themselves depend on other quantities, including corrections and correction factors for systematic effects, thereby leading to a complicated functional relationship f that may never be written down explicitly. Further, f may be determined experimentally or exist only as an algorithm that must be evaluated numerically. The function f as it appears in this Guide is to be interpreted in this broader context.

The set of input quantities $X_1,\dots,X_N$ may be categorized as:

- Quantities whose values and uncertainties are directly determined in the current measurement procedure. These values and uncertainties may be obtained from, for example, a single observation, repeated observations, or judgment based on experience, and may involve the determination of corrections to instrument readings and corrections for influence quantities, such as ambient temperature, barometric pressure and humidity;
- Quantities whose values and uncertainties are brought into the measurement procedure from external sources, such as quantities associated with calibrated measurement standards, certified reference materials and reference data obtained from handbooks.

An estimate of the measurand *Y*, denoted by *y*, is obtained from equation (1) using input estimates $x_1,\ldots,x_N$ fro the values of the *N* quantities $X_1,\ldots,X_N$. Thus the output estimate *y*, which is the result of the measurement, is given by

$$y = f\left(x_1, \ldots\ldots\ldots x_N\right) \quad (2)$$

The estimated standard deviation of the estimate *y*, termed combined standard uncertainty and denote by $u_c(y)$, is determined from the estimated standard deviation of each input estimate x_i, termed standard uncertainty and denoted by $u(x_i)$. Each input estimate x_i and its standard uncertainty $u(x_i)$ is obtained from a distribution of possible values of the input quantity X_i. This probability distribution may be frequency based, that is, based on a series of observations $X_{i,k}$ or X_i; or it may be a subjective, or *a pripori*, distribution. Type A evaluations of standard uncertainty components are founded on frequency distributions while Type B evaluations are founded on a priori distributions. It must be recognized that in both cases the distributions are models that are used to represent the state of our knowledge.

3.7. Type A evaluation of standard uncertainty

In most cases, the best available estimate of the expectation or expected value μ_q of a quantity *q* that varies randomly and for which *n* independent observations q_k have been obtained under the same conditions of measurement is the arithmetic mean or average $\bar{q}$ of the *n* observations:

$$\bar{q} = \frac{1}{n}\sum_{k=1}^{n} q_k \quad (3)$$

Thus, fro input quantity X_i estimated from *n* observations $X_{i,k}$, the sample mean $\bar{X}_i$ obtained from equation (3) is used as the input estimate x_i in equation (2) to determine the measurement result *y*; that is, $x_i = \bar{X}_i$. Those input estimates not evaluated from repeated observations must be obtained using other methods, such as those indicated up.

The individual observations q_k differ in value because of random variations in the influence quantities, or random effects. The variance of the observations, which estimates the variance σ^2 of the probability distribution of *q*, is obtained from

$$s^2(q_k) = \frac{1}{n-1}\sum_{k=1}^{n}(q_k - \overline{q})^2 \quad (4)$$

The sample estimate of variance $s^2(q_k)$ and its positive square root $s(q_k)$, termed the sample or experimental standard deviation, characterize the variability of the observed values q_k or their dispersion about their mean $\overline{q}$.

The best estimate of $\sigma^2(\overline{q}_k)$, the variance of the mean $\overline{q}$, is given by

$$s^2(\overline{q}) = \frac{s^2(q_k)}{n} \quad (5)$$

The variance $s^2(\overline{q}_k)$ and the experimental standard deviation of the mean $s(\overline{q}_k)$, equal to the positive square root of $s^2(\overline{q}_k)$, quantity how well $\overline{q}_k$ estimates the expectation μ_q of q and either one may be used as a measure of the uncertainty of $\overline{q}_k$.

Thus, for an input quantity X_i determined from n observations $X_{i,k}$, the standard uncertainty $u(x_i)$ of its estimate $x_i = \overline{X}_i$ is $u(\overline{X}_i) = s(\overline{X}_i)$, with the variance $s^2(\overline{X}_i)$ calculated according to equation (5). For convenience, $u^2(\overline{X}_i) = s^2(\overline{X}_i)$ and $u(\overline{X}_i) = s(\overline{X}_i)$ are sometimes referred to us, respectively, a Type A variance and a Type A standard uncertainty.

For well-characterized measurement procedures under statistical control, a combined or pooled sample variance s^2_p or pooled sample standard deviation s_p for the procedure may be available. In such cases the variance of the mean of *n* independent repeated observations is s^2_p/n and the standard uncertainty is $u = s_p/\sqrt{n}$.

Often the estimated value x_i of an input quantity X_i is obtained from a curve that has been fitted to experimental data by the method of least squares. The variance and resulting standard uncertainty of the fitted parameters characterizing the curve and of any predicted point can readily be calculated by well-known statistical procedures.

The degrees of freedom v_i of x_i and $u(x_i)$, equal to $n-1$ in the simple case where $x_i = \overline{X}_i$ and $u(\overline{X}_i) = s(\overline{X}_i)$ are calculated from n independent observations as last, should always be given when documenting Type A evaluations of uncertainty components.

If the random variations in observations of an input quantity are correlated, for example, in time, the mean and standard deviation of the mean as given lately may be inappropriate estimators of the desired statistics. In such cases, the observations should be analyzed using statistical methods specially designed to treat a random correlated series of measurements.

The above discussion of Type A evaluation of standard uncertainty is not meant to be exhaustive; there are many situations, some rather complex, that can be treated by statistical methods. An important example is the use of calibration design, often based on the method of last squares, to evaluate the uncertainties arising from both short- and long-term random variations in the results of comparisons of material artifacts of unknown value, such as gauge blocks and standards of mass, with reference standards of known value. In such comparatively simple measurement situations, components of uncertainty are frequently amenable to statistical evaluation by using designs consisting of nested sequences of measurements of the measurand for a number of different values of the quantities upon which it depends – a so-called analysis of variance.

3.8. Type B evaluation of standard uncertainty

For an estimate x_i of an input quantity X_i that has not been obtained from repeated observations, the estimated variance $u^2(x_i)$ or standard uncertainty $u(x_i)$ is evaluated by judgment using all relevant information on the possible variability of X_i. The pool of information may include previous measurement data, experience with or general knowledge of the behavior and properties of relevant materials and instruments, manufacturer's specifications, data provided in calibration and other certificates, and uncertainties assigned to reference data taken from handbooks. For convenience, $u^2(x_i)$ and $u(x_i)$ estimated in this way are sometimes referred to as, respectively, a Type B variance and a Type B standard uncertainty.

The proper use of the pool of available information for a Type B evaluation of standard uncertainty calls for insight based on experience and general knowledge, but is a skill that a Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation, especially in a measurement situation where a Type A evaluation is based on a comparatively small number of statistically independent observations.

If the estimate x_i is taken from a manufacturer's specification, calibration certificate, handbook, or other source, and its uncertainty is given as a multiple of a standard deviation, the standard uncertainty $u(x_i)$ may be taken equal to the quoted value divided by the multiplier, and the estimated variance $u^2(x_i)$ may be taken equal to the square of the quotient.

The uncertainty of x_i may be given as a 90, 95, or 99 percent confidence interval rather than as a multiple of a standard deviation. Unless otherwise indicated, one may assume that a normal distribution was used to calculate the intervals, and recover the standard uncertainty for x_i by dividing the given confidence interval by the appropriate factor for the normal distribution. The factors corresponding to the above three intervals are 1,64; 1,96; and 2,58.

Based on the available information, it may be possible to state, "there is a fifty-fifty chance that the value of the input quantity X_i lies in the range a_- to a_+" (in other words, the probability that X_i lies within this range is 0,5 or 50 percent). If the best estimate x_i of X_i can be taken to be the midpoint of the range, if the half width of the range is denoted by $(a_+ - a_-)/2 = a$, and if it can be assumed that the distribution of possible values of X_i is approximately normal, then one may take $u(x_i) = 1{,}48a$ and $u^2(x_i) = (1{,}48a)^2$, since for a normal distribution with expectation μ and standard deviation σ, the interval $\mu \pm \sigma/1{,}48$ encompasses 50 percent of the distribution.

It may be possible to evaluate $u(x_i)$ more or less directly. For example, the available information may allow one to state, "there is about a 2 out of 3 chance that the value of X_i lies in the range a_- *to* a_+" (in other words, the probability that X_i lies within this range is about 0,67). If the range is symmetric about the estimate x_i with half width $(a_+ - a_-)/2 = a$, and it can be assumed that the possible values of X_i are approximately normally distributed, then one may reasonably take $u(x_i) = a$ and $u^2(x_i) = a^2$, since for a normal distribution with expectation μ and standard deviation σ, the interval $\mu \pm \sigma$ encompasses 68,3 percent of the distribution.

In other cases it may only be possible to estimate bounds (upper and lower limits) for X_i, in particular, to state that "the probability that the value of X_i lies within the range a_- *to* a_+ for all practical purposes is equal to 1 and the probability that X_i lies outside this range is essentially 0." If there is no specific knowledge about the possible values of X_i within the range, it can only be assumed that it is equally probable for X_i to lie anywhere within it. Then x_i, the expectation or expected value of X_i, is the midpoint of the range: $(a_+ - a_-)/2 = x_i$, with variance

$$u^2(x_i) = (a_+ - a_-)^2 / 12 \qquad (6)$$

If the difference between the bounds, $a_+ - a_-$, is denoted by 2a, then equation (6) becomes

$$u^2(x_i) = a^2/3 \qquad (7)$$

The upper and lower bounds a_+ and a_- for the input quantity X_i may not be symmetric with respect to its best estimate x_i; more specifically, if the lower bound is written as $a_- = x_i - b_-$ and the upper bound as $a_+ = x_i + b_+$, then $b_- \neq b_+$. Since in this case x_i (assumed to be the expectation of X_i) is not at the centre of the interval a_- to a_+, the probability distribution of X_i cannot be uniform throughout the interval. However, there may not be enough information available to choose an appropriate distribution; different models will lead to different expressions for the variance. In the absence of such information the simplest approximation is

$$u^2(x_i) = \frac{(b_+ + b_-)}{12} = \frac{(a_+ - a_-)}{12} \qquad (8)$$

Which is the variance of a rectangular distribution with full width $b_+ + b_-$. As it was mentioned, because there was no specific knowledge about the possible values of X_i within its estimated bounds a_- *to* a_+, it was assumed that it was equally probable for X_i to take any value within those bounds, with zero probability of being outside them. Such step function discontinuities in a probability distribution are unphysical. In many cases it is more realistic to expect that values near the bounds are less likely than those near the midpoint. It is then reasonable to replace the symmetric rectangular distribution with a symmetric trapezoidal distribution having equal sloping sides (an isosceles trapezoid), a based of width $a_+ + a_- = 2a$, and a top of width $2\alpha\beta$, where $0 \leq \beta \leq 1$. As $\beta \to 1$ this trapezoidal distribution approaches the rectangular distribution, while for $\beta = 0$ it is a triangular distribution. Assuming such a trapezoidal distribution for X_i, the expectation of X_i is $(a_+ - a_-)/2 = x_i$, and its variance is

$$u^2(x_i) = a^2(1+\beta^2)/6 \tag{9a}$$

Which becomes for the triangular distribution, $\beta = 0$,

$$u^2(x_i) = a^2/6 \tag{9b}$$

The above discussion of Type B evaluation of standard uncertainty is meant only to be indicative. Further, the need to base the evaluation of uncertainty on quantitative data to the maximum possible extent, should always be borne in mind.

3.9. Graphical illustration of evaluating standard uncertainty

Figure (1) represents graphically the estimation of the value of an input quantity X_i and the evaluation of the uncertainty of that estimate from the unknown distribution of possible measured values of X_i, or probability distribution of X_i, that is sampled by means of repeated observations.

In figure (1a) the input quantity X_i is assumed to be a temperature t measured in °C and its unknown distribution a normal distribution with expectation $\mu_t = 100°C$ and standard deviation $\sigma = 1{,}5°C$. Its probability density function is then

$$p(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-(t-\mu_t)^2/2\sigma^2\right]$$

Shown in figure (1b) is a histogram of $n = 20$ experimental observations t_k of the temperature t assumed to be determined by the distribution of figure (1a). To obtain the histogram, the 20 observations or samples, whose values are given in table 1, are grouped into intervals 1°C wide.

The arithmetic mean or average $\bar{t}$ of the $n = 20$ observations calculated according to equation (3) is $\bar{t} = 100{,}145°C \approx 100{,}14°C$, and is assumed to be the best estimate of the expectation μ_t of t based on the available data. The sample or experimental standard deviation $s(t_k)$ calculated from equation (4) is $s(t_k) = 1{,}489°C \sim 1{,}49°C$; and the experimental standard deviation of the mean $s(\bar{t})$ calculated from equation (5), which is the standard uncertainty $u(\bar{t})$ of the mean $\bar{t}$, is $u(\bar{t}) = s(\bar{t}) = s(t_k)/\sqrt{20} = 0{,}333°C \approx 0{,}33°C$.

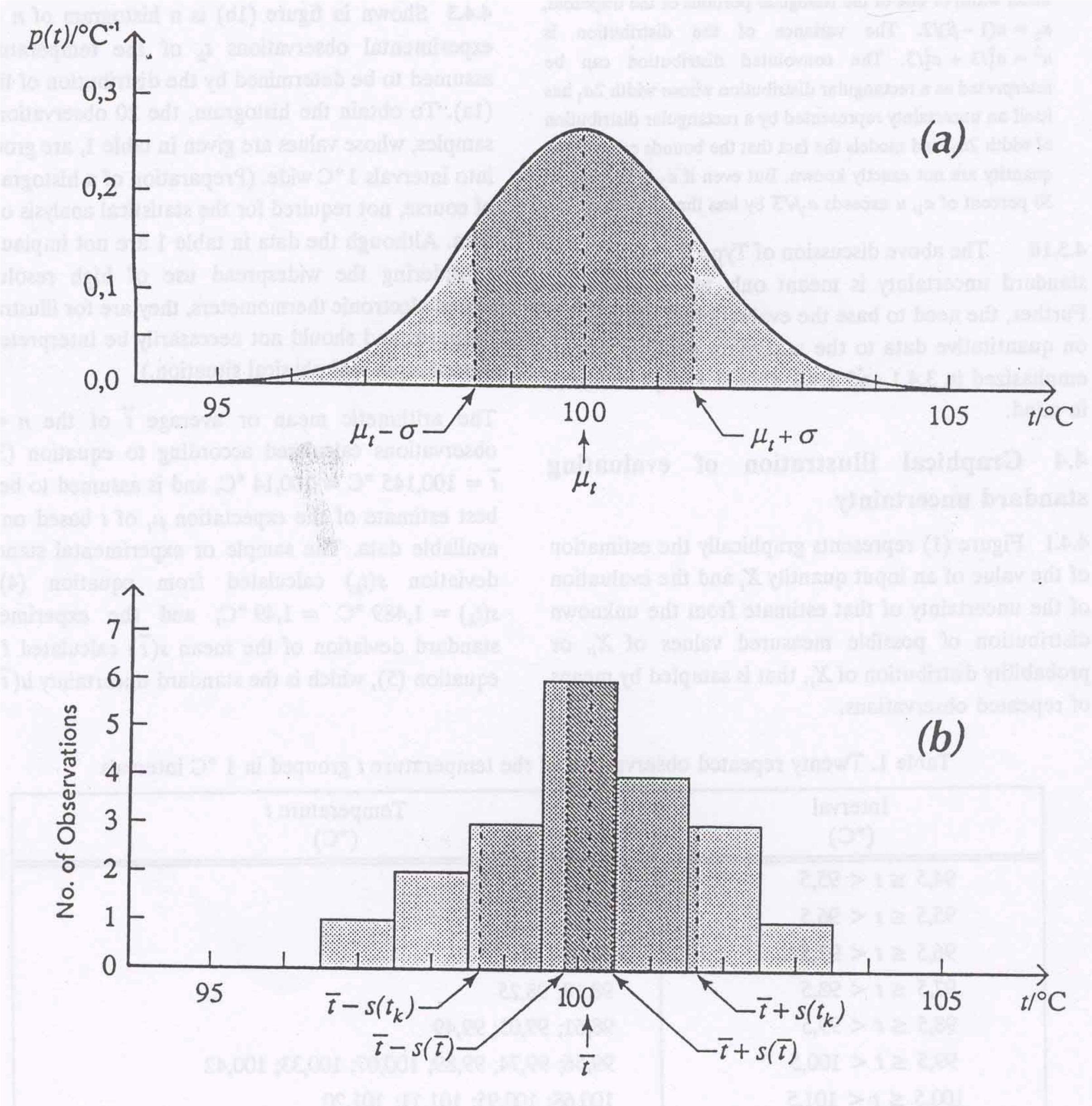

Figure 1. Graphical illustration of evaluating the standard uncertainty of an input quantity from repeated observations

Figure (2) represents graphically the estimation of the value of an input quantity X_i and the evaluation of the uncertainty of that estimate from an *a priori* distribution of possible values of X_i, or probability distribution of X_i, based on all available information. For the particular case shown, the input quantity is again assumed to be a temperature t.

For the case illustrated in figure (2a), it is assumed that very little information is available about the input quantity t and that all one can do is suppose that t is described by a symmetric rectangular a priori probability distribution of lower bound $a_- = 96°C$, upper bound $a_+ = 104°C$, and thus half width $a = (a_+ - a_-)/2 = 4°C$. The probability density function of t is then

$p(t) = 1/2a, a_- \le t \le a_+$

$p(t) = 0 \ldots otherwise$

As was indicated, the best estimate of t is its expectations $\mu_t = (a_+ + a_-)/2 = 100°C$. The standard uncertainty of this estimate is $u(\mu_t) = a/\sqrt{3} \approx 2{,}3°C$, which follows from equation (7).

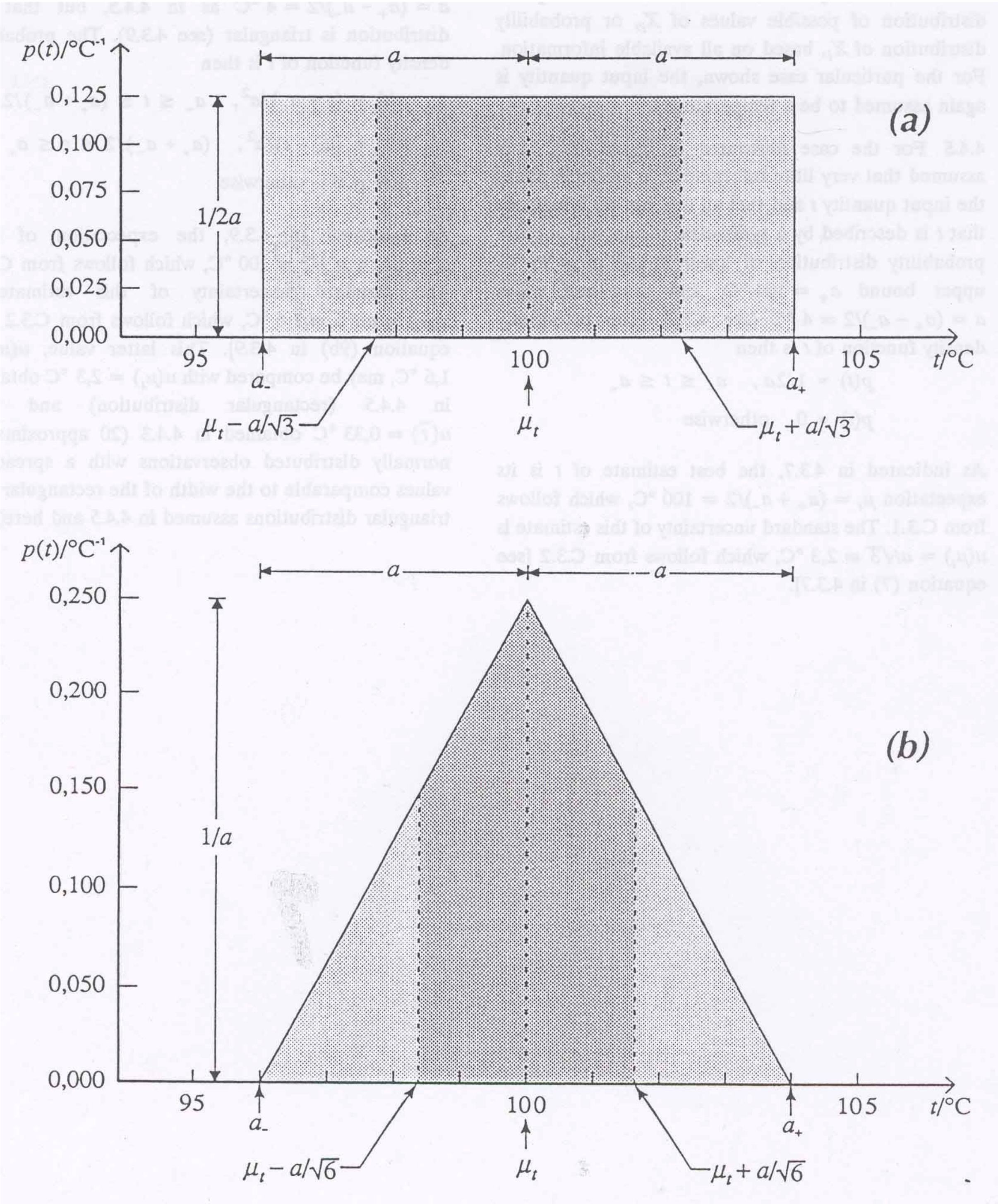

Figure 2. Graphical illustration of evaluating the standard uncertainty of an input quantity from an a priori distribution

For the case illustrated in figure (2b), it is assumed that the available information concerning t is less limited and that t can be described by a symmetric a priori probability distribution of the same lower bound $a_- = 96°C$, the same upper bound $a_+ = 104°C$, and thus the same half width $a = (a_+ - a_-)/2 = 4°C$, but that the distribution is triangular. The probability density function of t is then

$$p(t) = (t - a_-)/a^2, a_- \le t \le (a_+ + a_-)/2$$
$$p(t) = (a_+ - t)/a^2, (a_+ + a_-)/2 \le t \le a_+$$
$$p(t) = 0 \ldots otherwise$$

As indicated lately, the expectation of t is $a = (a_+ - a_-)/2 = 100°C$, which follows from last mentioned. The standard uncertainty of this estimate is $u(\mu_t) = a/\sqrt{6} \approx 1{,}6°C$.. This latter value $u(\mu_t) = 1{,}6°C$, may be compared with $u(\mu_t) = 2{,}3°C$, and with $u(\bar{t}) = 0{,}33°C$ obtained see above (20 approximately normally distributed observations with a spread of values comparable to the width of the rectangular and triangular distributions assumed above and here).

3.10. Determining combined standard uncertainty

3.10.1. Uncorrelated input quantities

The case where two or more input quantities are related, that is, is interdependent or correlated, is discussed in next chapter.

The total uncertainty of *y*, where *y* is the estimate of the measurand *Y* and thus the result of the measurement, is obtained by appropriately combining the standard uncertainties of the input estimates $x_1, x_2, \ldots, x_N$. This combined standard uncertainty of the estimate *y* is designed by $u_c(y)$.

The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$ obtained from

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad (10)$$

Where $u(x_i)$ is a standard uncertainty evaluated as described above for *Type A evaluation* or for *Type B evaluation*. The uncertainty $u_c(y)$ is an estimate of the standard deviation of the distribution of possible values, or probability distribution, of *y*.

The quantities $\partial f / \partial x_i$ are the partial derivatives of $y = f(x_1, \ldots, x_N)$, equation (2). These derivatives, often referred to as sensitivity coefficients, describe how the output estimate *y* varies with changes in the values of the input estimates $x_i, \ldots, x_N$. In particular, the change in *y* produced by a small change Δx_i in the input estimate x_i is given by $(\Delta y)_i = (\partial f / \partial x_i)(\Delta x_i)$. If this change is generated by the standard uncertainty of the estimate x_i, the corresponding uncertainty in *y* is $u_i(y) = (\partial f / \partial x_i) u(x_i)$. The combined variance $u_c^2(y)$ can therefore be viewed as a sum of terms representing the variance of the output estimate y generated by the variance (or standard uncertainty) of each input estimate x_i. This suggests writing equation (10) as

$$u_c^2(y) = \sum_{i=1}^{N} \left[c_i u(x_i)\right]^2 \equiv \sum_{i=1}^{N} u_i^2(y) \qquad (11a)$$

Where

$$c_i \equiv \partial f / \partial x_i, u_i(y) \equiv c_i u(x_i) \qquad (11b)$$

Instead of being calculated from the function f, the sensitivity coefficients $\partial f / \partial x_i$ are sometimes determined experimentally by measuring the change in y produced by a change in x_i. In this case the knowledge of f (or a portion of it) is correspondingly reduced to an empirical first-order Taylor series expansion based on the measured sensitivity coefficients.

If equation (1) for the measurand Y is expanded about nominal values $X_{i,0}$ of the input quantities X_i, then, to first order, $Y = Y_0 + c_1\delta_1 + \ldots + c_N\delta_N$, where $Y_0 = f(X_{1,0}, \ldots, X_{N,0})$, $c_i = (\partial f / \partial X_i)$ evaluated at $X_i = X_{i,0}$, and $\delta_i = X_i - X_{i,0}$. Thus, for the purposes of an analysis of uncertainty, a measurand can usually be expressed as a linear function of its variables by transforming its input quantities from X_i to δ_i.

If Y is of form $Y = cX_1^{P_1} X_2^{P_2} \ldots X_N^{P_N}$ and the exponents p_i are known positive or negative numbers having negligible uncertainties, the combined variance, equation (10), can be expressed as

$$\left[u_c(y)/Y\right]^2 = \sum_{i=1}^{N} \left[p_i u(x_i)/x_i\right]^2 \qquad (12)$$

This is of the same form as equation (11a) but with the combined standard uncertainty $u_c(y)$ expressed as a relative combined standard uncertainty $u_c(y)/y$ and the standard uncertainty $u(x_i)$ of each input estimate expressed as a relative standard uncertainty $u(x_i)/x_i$.

3.10.2. Correlated input quantities

Equation (10) and those derived from it such as equations (11) and (12) are valid only if the input quantities X_i are independent or uncorrelated. If any of the X_i is significantly correlated, the correlation between any two input quantities X_i and X_j should be distinguished from the correlation between their estimates x_i and x_j. Even when there is no real correlation between X_i and X_j, the correlation between the estimates x_i and x_j must be included in a fully consistent treatment of the uncertainty of those estimates.

When the input quantities or their estimates are correlated, the appropriate expression for the combined variance $u_c^2(y)$ of the result of a measurement is

$$u_c^2(y) = \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i,x_j) \quad (13)$$

Where x_i and x_j are the estimates of X_i and X_j and $u(x_i,x_j) = u(x_j,x_i)$ is the estimated covariance of x_i and x_j. The degree of correlation between x_i and x_j is characterized by the estimated correlation coefficient

$$r(x_i,x_j) = \frac{u(x_i,x_j)}{u(x_i)u(x_j)} \quad (14)$$

Where $r(x_i,x_j) = r(x_j,x_i)$, and $-1 \le r(x_i,x_j) \le +1$. If the estimates x_i and x_j are independent, $r(x_i,x_j) = 0$, and a change in one does not imply a change in the order.

In terms of correlation coefficients, which are more readily interpreted than covariances, the covariance term of equation (13) may be written as

$$2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i)u(x_j)r(x_i,x_j) \quad (15)$$

Thus equation (13) becomes, with the aid of equation (11b),

$$u_c^2(y) = \sum_{i=1}^{N}u_i^2(y) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}u_i(y)u_j(y)r(x_i,x_j) \quad (16)$$

In general, an estimate of the covariance of two arithmetic means $\bar{q}$ and $\bar{r}$ that estimate the expectations μ_q and μ_r of two random varying quantities q and r and that are obtained from n repeated simultaneous observations is given by

$$s(\bar{q},\bar{r}) = \frac{1}{n(n-1)}\sum_{k=1}^{n}(q_k - \bar{q})(r_k - \bar{r}) \quad (17)$$

Where q_k and r_k are the individual observations of the quantities q and r, and $\bar{q}$ and $\bar{r}$ are calculated from the observations according to equation (3). If in fact the observations are uncorrelated, the calculated covariance is expected to be near 0.

Thus, the estimated covariance of two correlated input quantities X_i and X_j that are estimated by means $\bar{X}_i$ and $\bar{X}_j$ determined from pairs of simultaneous observations is given by $u(x_i,x_j) = u(\bar{X}_i,\bar{X}_j) = s(\bar{X}_i,\bar{X}_j)$, with $s(\bar{X}_i,\bar{X}_j)$ calculated according to equation (17). This application of equation (17) may be viewed as a Type A evaluation of covariance. The estimated correlation coefficient of $\bar{X}_i$ and $\bar{X}_j$ is obtained from equation (14): $r(\bar{X}_i,\bar{X}_j) = s(\bar{X}_i,\bar{X}_j)/s(\bar{X}_i)s(\bar{X}_j)$.

Covariances cannot be ignored if present; they should be evaluated experimentally if feasible by varying the correlated input quantities or by using all available information on the correlated variability of the quantities in question.

Insight based on past experience and general knowledge is especially required when estimating the degree of correlation between input quantities arising from the effect of common influence quantities such as ambient temperature, barometric pressure, and humidity. In many cases, however, the effects of influence quantities are sufficiently independent that the affected input quantities can be assumed to be uncorrelated.

A significant correlation may exist between two input quantities if the same measuring instrument, physical measurement standard, or reference datum having a significant standard uncertainty is used in their determination. For example, if a temperature correction required in the estimation of the value of input quantity X_i is obtained using a certain thermometer, and a similar temperature correction required in the estimation of input quantity X_j has also been obtained using the same thermometer, the two input quantities could be significantly correlated.

3.11. Determining expanded uncertainty

3.11.1. Expanded uncertainty

The additional measure of uncertainty that meets the requirement of providing an interval of the kind indicated above, or confidence interval, is termed *expanded uncertainty* and is denoted by U. The expanded uncertainty U is obtained by multiplying the combined standard uncertainty $u_c(y)$ by a *coverage factor* k:

$$U = ku_c(y) \tag{18}$$

The result of a measurement is then conveniently expressed as $Y = y \pm U$, which is interpreted to mean that the best estimate of the value attributable to the measurand Y is y, and that the interval defined by $y - U$ to $y + U$ contains with a high level of confidence p the values that could reasonably be attributed to Y. Such a confidence interval is also expressed as $y - U \leq Y \leq y + U$.

3.11.2. Choosing a coverage factor

The value of the coverage factor k is chosen on the basis of the desired level of confidence to be associated with the interval $y - U$ to $y + U$. In general, k will be in the range from 2 to 3. However, for special applications k may be outside this range.

Ideally, one would like to be able to choose a specific value of the coverage factor k that would provide an interval $Y = y \pm U = y \pm ku_c(y)$ corresponding to a dwell-defined level of confidence p, or equivalently, for a given value of k, one would like to be able to state unequivocally the level of confidence associated with that interval. However, this is not easy to do in practice because it requires full knowledge of the probability distribution of Y. In most cases, all that is available is the estimate y and its combined standard uncertainty (estimated standard deviation), $u_c(y)$. Although these parameters are of critical importance, they are by themselves insufficient for the purpose of establishing intervals having exactly known levels of confidence.

Recommendation INC-1 (1980) does not specify how the relation between *k* and *p* should be established. Although the method is not overly difficult to implement, that a reasonably adequate general approach to choosing a value of the coverage factor *k* and stating at level of confidence to be associated with the interval $Y = y \pm U = y \pm ku_c(y)$, is to recognize that the distribution of $(y - Y)/u_c(y)$ is approximately normal and to take $k = 2$ to obtain an interval having a level of confidence of approximately *95 percent*, or to take $k = 3$ to obtain an interval having a level of confidence of approximately *99 percent*. Although this is an elementary approach, nevertheless it should be applicable to a broad range of practical measurement situations.

3.12. Summary of procedures for the expression of uncertainty

The procedures to be followed for evaluating and expressing the uncertainty of the result of a measurement as presented in this guide may be summarized as follows:

1. Express mathematically the relationship between the measurand Y and the input quantities X_i on which *Y* depends: $Y = f(X_1, \ldots, X_N)$.

2. Determine x_i, the estimated value of input quantity X_i, either on basis of the statistical analysis of a series of observations or by other means. Each input estimate x_i is to include corrections for all known systematic effects that significantly influence the estimate *y* of the measurand *Y*, which is the result of the measurement.

3. Evaluate the *standard uncertainty* $u(x_i)$ of each input estimate x_i. For an input estimate obtained from the statistical analysis of a series of observations, the standard uncertainty $u(x_i)$ is evaluated as described above (*Type A evaluation of standard uncertainty*). For an input estimate obtained by other means, the standard uncertainty $u(x_i)$ is too evaluated as described above (*Type B evaluation of standard uncertainty*).

4. If the values of any input quantities are correlated, evaluate their covariances.

5. Calculate the estimate *y* of the measurand *Y* from the functional relationship *f* using for the input quantities X_i the estimates x_i obtained in step 2.

6. Determine the *combined standard uncertainty* $u_c(y)$ of the measurement result *y* from the standard uncertainties and covariances of the input estimates. If the measurement procedure determines simultaneously more than one output quantity, calculate their covariances.

7. If it is required to give an *expanded uncertainty U*, whose purpose is to provide an interval *y – U to y + U* within which the values that could reasonably be attributed to the measurand *Y* may be expected to lie with a high level of confidence, multiply the combined standard uncertainty $u_c(y)$ by a *coverage factor k*, typically in the range *2 to 3*, to obtain $U = ku_c(y)$. Select *k* on the basis of the desired level of confidence to be associated with the interval.

8. Report the result of measurement *y* together with its standard uncertainty $u_c(y)$ or expanded uncertainty *U* as indicated above, and describe how they were obtained.

4. STOCHASTIC PROCESSES

4.1. Process types

Analyze of a dynamics system involves analyze of its dynamics characters as a complex and analyze input- and output-quantities. As the input quantities can be forces, force-pair, pressures etc., as the output quantities can be loading, stress, strain, deformation, deviation, and acceleration – where mostly each of them is a function of time. All of them that are variable in time are possible to sum into common conception – process.
We can divided them as follows (fig. 1.1):
a) Deterministic,
b) Stochastic,
c) Hybrid,
d) Inhomogeneous.

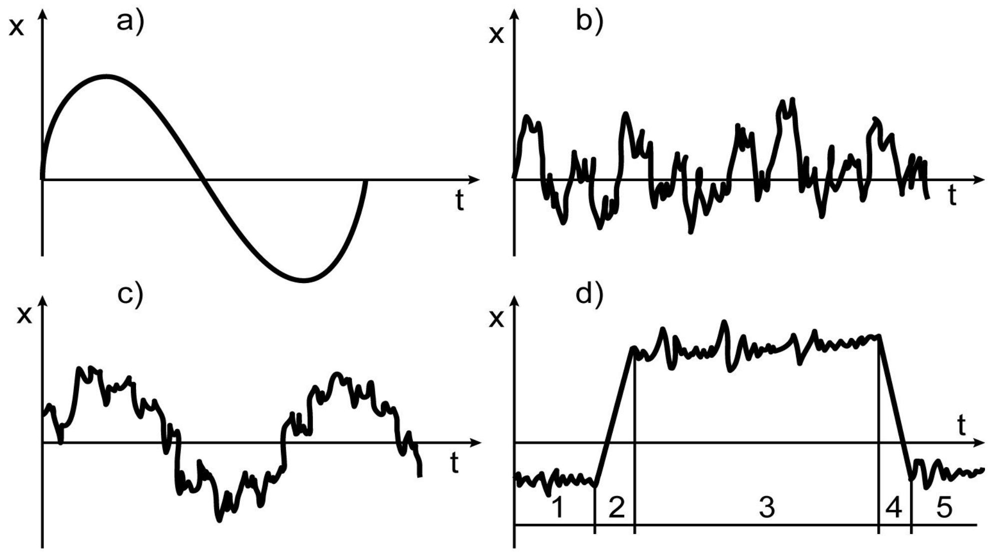

Figure. 4.1

Ad a) it is possible to determine their value for deterministic processes at any point from the system of differential equations and known initial conditions. While repeating of the experimental measuring under the same conditions the deterministic process has been going always in the same way.

Ad b) there is no possibility to determine their values for stochastic process. One way to specify them is by using of mathematical statistics. While repeating of the experimental measuring under the same conditions the stochastic process hasn't been going in the same way. These characteristics are different from each other.

Ad c) in practice, there are mostly both of them in the equal rate.

Ad d) analyze of an inhomogeneous process is able to do only for its homogeneous parts that are connected with function of transitional character. Some of them are non-stationary (resp. inhomogeneous) so much that there is too difficult to specify them mathematically.

4.2. Frequency analyze

For some tasks or solution of any problems is better to transform the time-course of process to the frequency-course, it means to replace it by row of its frequency parts. These operation calls frequencies analyze. These frequency components provide important information mainly about the sources of oscillating. Excitation of an arbitrary time-course (inclusive of impression) is possible to makes up for sequence of basic peeks of different frequencies and amplitudes. It is possible after that to find out a response for arbitrary excitation for linear systems. There is also possible to compare possibilities of rising of the dangerous tuned states for different process activities from comparison of excitation's frequency analyze, from the change of frequency components in processing we can explore also rise and development of failure. These facts are often used for operational diagnostics.

It is called harmonic analyze for periodical processes to determine amplitudes and also angle phase of their components. For that it is used so-called Fourier's sequence. Acquired spectra are discreet. In stochastic processes are often used Fourier's integral transformation for the same purpose. Acquired spectra are continuous. To determine of incidence of particular sources for mechanical oscillation upon the mechanical system are very important power spectrums and also power spectral densities.

4.2.1. Fourier sequence

Each and every periodical process is possible to interpret as a superposition of many elemental sin and cosine functions.

Fourier sequence for given function *x(t)* (that answers Dirichlet's conditions about continuity of the function in the time interval) is infinite row

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega t) + \sum_{k=1}^{\infty} b_k \sin(k\omega t)$$

Where

$$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega t)\,dt$$

k = 0, 1, 2, …

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega t)\,dt$$

k = 1, 2, …

T … basic period

$\omega = \frac{2\pi}{T}$ …. Basic angle frequency

More convenient is representation Fourier row in the form

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega t + \psi_k)$$

k = 1, 2, …

$$A_0 = \frac{a_0}{2} \qquad A_k = \sqrt{a_k^2 + b_k^2} \qquad \psi_k = arctg\frac{-b_k}{a_k}$$

To determine of coefficients A_k and ψ_k is called ***harmonic analyze***. Particular elemental running $A_k cos(k\omega t + \psi_k)$ are called ***harmonic component***. Quantity A_k is then ***amplitude*** and ψ_k ***phase of harmonic component***. Amplitude allocate to the frequency is called ***spectrum of amplitudes***; phase angle allocate to the frequency is ***spectrum of phase angles***.

With regard to that coefficients A_k are determined only by values of functions in the interval of periodicity (0, T), is possible to create Fourier row to the function that isn't not periodical, but that is defined in the interval (0, T) or that is not a zero and lies into the finite interval.

Power spectrum of the periodical signal represents the powers of every harmonic components for resistance 1 Ω , it means the sequence of the values $A_k^2 / 2$, where A_k are amplitudes of each harmonic components. This spectrum describes the frequency layout of the power of periodical signal. Total sum of all components gives the complete signal power.

For no periodic process we are speaking about energy spectrum that is used up by so called energy spectral density.

4.2.2. Fourier transform

In Fourier transform (briefly FT) is the original signal replaced by sequences of harmonic functions with different frequencies and phase so as the sum of these particular waves was the original signal. Relation defines this direct FT

$$X(f) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} x(t).exp(-j2\pi ft) dt$$

Complex function $X(f)$ is then called Fourier transformation of real function $x(t)$.

There is also possibility achieve back the original by ***inverse FT*** being defined with following relation

$$x(t) = \int_{-\infty}^{\infty} X(f).exp(j2\pi ft) df$$

In the items form

$$F\{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t).cos(2\pi ft) dt - j\int_{-\infty}^{\infty} x(t).sin(2\pi ft) dt$$

$$X(f) = Re\{X(f)\} - j.Im\{X(f)\}$$

If the function x(t) is even, it is if holds x(-t) = x(t), then the function x(t).cos(2πft) is also even and function x(t).sin(2πft) is odd; at the mentioned interval is then

$$\int_{-\infty}^{\infty} x(t).sin(2\pi ft)dt = 0$$

and so

$$X(f) = \int_{-\infty}^{\infty} x(t).cos(2\pi ft)dt = 2\int_{0}^{\infty} x(t).cos(2\pi ft)dt$$

Inverse Fourier transformation of the even function x(t) can be defined analogically

$$x(t) = 2\int_{0}^{\infty} X(f).cos(2\pi ft)df$$

For equality of the energy in time- and frequency-level must hold true the relation

$$\int_{-\infty}^{\infty} \left[x(t)\right]^2 dt = \int_{-\infty}^{\infty} \left|X(f)\right|^2 df$$

For processing of the stochastic processes at the PC it is based on discrete parts. For that it is used ***discrete Fourier transformation*** (DFT). In this case the original and also the image has discrete character. The solution is more time-difficult - it is assumed to do N^2 complex multiplying and N^2 complex add (where N is the number of samples). This activity can be effectively shortened by algorithm ***fast Fourier transformation*** (FFT).

4.3. Classification of the stochastic processes

Using of theories of the stochastic processes in the branch of dynamics, rigidity and lifetime of constructions is mainly significant because the best part of quantities has a stochastic character.

In practice, there is very often situation where the variables can be changed during the process. The result of a stochastic attempt is then time function that has the stochastic values. This process can be characterized as a set of possible different time functions so called ***realizations*** of the stochastic process.

4.3.1. Characters of the set of realization

It is assumed a cut through of the stochastic process X(t) at the time t = t_1 (fig. 4.2)

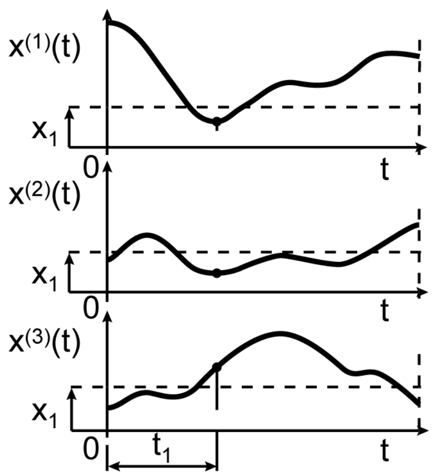

fig. 4.2

After that it can be defined probability

$$P\left[X\left(t_1\right) \le x_1\right] = F\left(x_1, t_1\right)$$

This function of two variables x_1, t_1 is called one-sized distribution function. And the one-sized density of probability is

$$f\left(x_1, t_1\right) = \frac{\partial F\left(x_1, t_1\right)}{\partial x_1}$$

Similarly there is possible to defined two-sized distribution function and also two-sized density of probability (fig. 4.3):

$$P\left[X\left(t_1\right) \le x_1; X\left(t_2\right) \le x_2\right] = F\left(x_1, t_1; x_2, t_2\right)$$

$$f\left(x_1, t_1; x_2, t_2\right) = \frac{\partial^2 F\left(x_1, t_1; x_2, t_2\right)}{\partial x_1 \partial x_2}$$

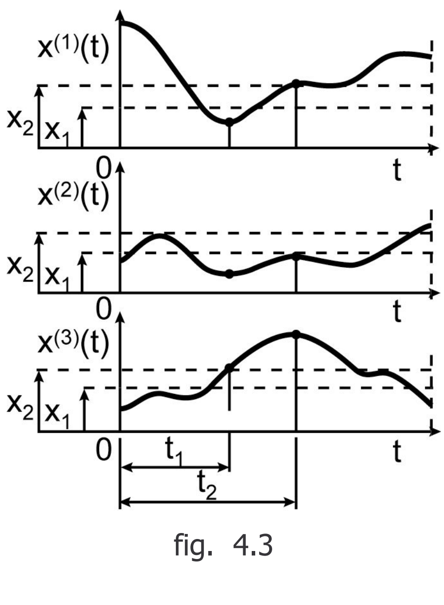

fig. 4.3

4.3.2. Stationary process

Narrower category of the stochastic processes is stationary processes. The stationary condition is possible to set in a different way:

a) As a stationary process we can call that one, which has the density of probability of arbitrary order, is not changed while changing of start point. So for example for n-th order is

$$f\left(x_1,x_2,\ldots x_n;t_1,t_2,\ldots t_n\right)=f\left(x_1,x_2,\ldots x_n;t_1+\tau,t_2+\tau,\ldots t_n+\tau\right)$$

These densities of probability are time-invariant.

The processes that satisfy this condition are called ***stationary at a short meaning*** or also ***strongly stationary***.

b) As a stationary process we can also call that one, which has the one- and two-sized density of probability are time-invariant. These processes are called ***stationary at a wider meaning*** or ***weakly stationary***. We will deal with them.

Here are:

- One-sized density of probability time t, invariant
- Two-sized density of probability is beside of arguments x_1 and x_2 function only $t_2 - t_1 = \tau$.

4.3.3. Ergodical processes

Ergodical processes are the main part of the stationary processes.

Briefly said that the process is ergodical if each and every alone realization of the stochastic function is as a plenipotentiary of the whole set of possible realization. One realization with sufficient length can replace the set of realization with the same length (as it is at fig. 4.4a, and not so like it is at fig. 4.4b).

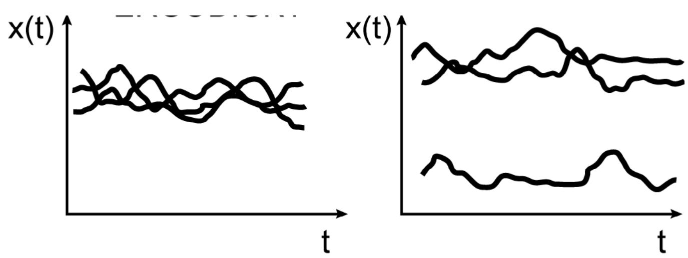

fig. 4.4a,b

Ergodicity and stationary of the stochastic process are two independent characteristics. Necessary condition of ergodicity is stationary, but it is not a condition sufficient.

Practically, the stochastic process is ergodical, if this process is stationary in widely meaning and its mean value in time and time-correlated function are for every realization the same. The results of analysis show that the most of the processes can be assumed as an ergodical.

While solving a particular task or problem we do not know if this process meets the condition of stationary. Then is possible to continue with following way. (fig. 4.5):

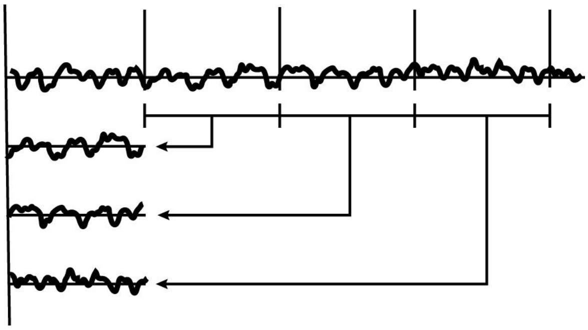

fig. 4.5

1. We can presume the stationary and ergodicity of the analyzed process,
2. We divide the whole entry to the number of files,
3. Every file is assumed as ergodical and is possible to assessed its characteristics,
4. If there are any differences of the mean value and variance for every file statistically insignificant we can come to a conclusion that the process is stationary. If the process is not stationary we have to use another method.

4.4. Characteristics of the ergodical processes

4.4.1. Characteristics in the time domain

4.4.1.1. Density of probability

It is presumed the entry of the process realization (fig. 4.6).

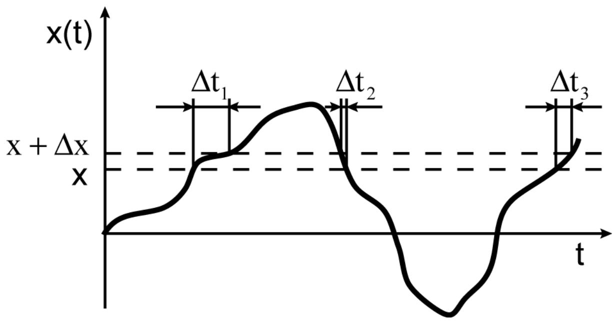

fig. 4.6

After that the probability that *x(t)* will lie between the limits from *x* to *(x + Δx)* and it is given by rate T_x / T, where T_x is the addition of all t_i :

$$P\left[x < x(t) \leq x + \Delta x\right] = lim_{T\to\infty}\frac{T_x}{T}$$

Then the density of probability is

$$f(x) = lim_{\Delta x\to 0}\frac{P\left[x < x(t) \leq x + \Delta x\right]}{\Delta x}$$

For small Δx is

$$f(x) = \frac{1}{\Delta x}\left[lim_{T\to\infty} \frac{T_x}{T} \right]$$

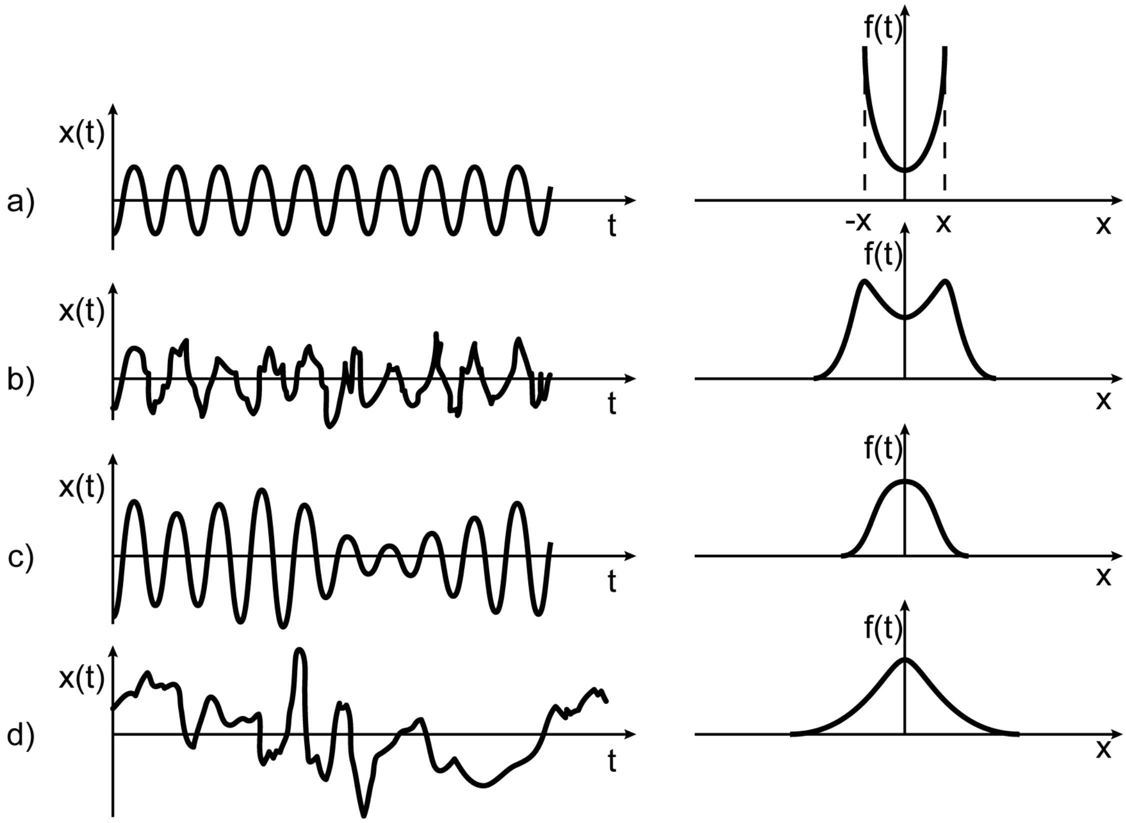

fig. 4.7

At the fig. 4.7 are shown:
a) Harmonic process,
b) Harmonic process with added random sound,
c) Narrow-zone stochastic process,
d) Broad-zone stochastic process.

Time value of the moment

The first general moment in time (time mean value)

$$m_1\left[x(t)\right] = lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\,dt = \overline{x}$$

The second general moment in time is

$$m_2\left[x(t)\right] = lim_{T\to\infty} \frac{1}{T}\int_0^\infty \left[x(t)\right]^2 dt = \psi_x^2$$

And effective value

$$x_{ef} = +\sqrt{m_2\left[x(t)\right]}$$

Time variance (second central moment in time)

$$\mu_2\left[x(t)\right] = lim_{T\to\infty}\frac{1}{T}\int_0^T\left[x(t)-\overline{x}\right]^2 dt = \sigma_x^2 = \psi_x^2 - \overline{x}^2$$

And also it is possible to define the other moments in this way.

Asymmetry factor

$$As = \frac{\mu_3}{\left(\mu_2\right)^{3/2}}$$

for *As > 0* is the mode lower than the mean value
for *As < 0* is the mode bigger than the mean value

Excess factor

$$Ex = \frac{\mu_4}{\left(\mu_2\right)^2} - 3$$

For *Ex > 0* is the layout of the density of probability „more excessive" than Gauss's.

For *Ex < 0* is the layout of the density of probability „more flatter" than Gauss's.

In practical study of these processes at PC is connected function replaced by discrete values received from sampling in time points t_k , where $k = 1, 2, \dots m$ (fig. 4.8).

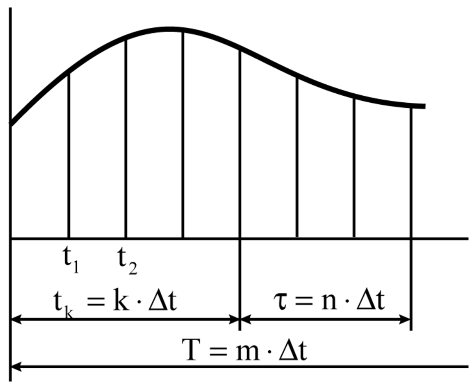

fig. 4.8

4.4.1.2. (Time) auto-correlative a auto-covariance function

In the fig. 4.9 are shown pictures of stochastic processes, where can be seen that as the mean value so the variance does not refer the inner state of the process.

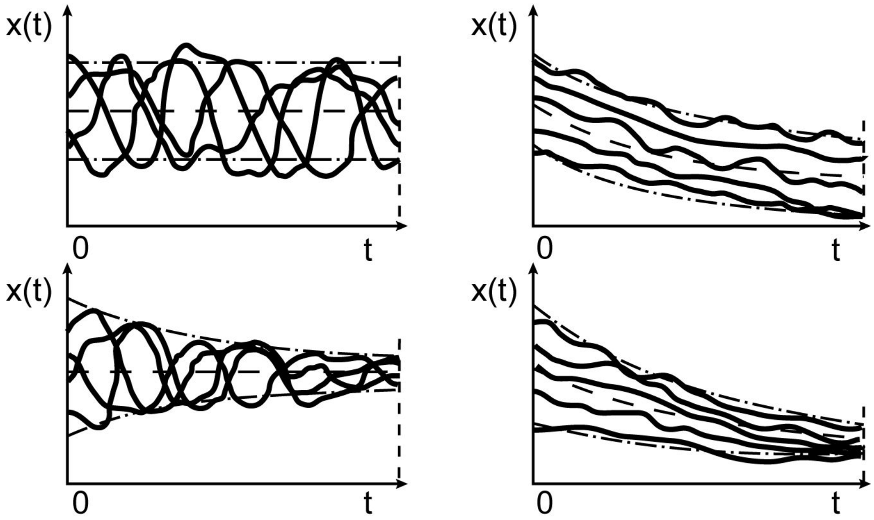

fig. 4.9

For that are often used two moments received from two cuts in time points t_1 , t_2 – auto-correlative function $R_{xx}(\tau)$ and auto-covariance function $K_{xx}(\tau)$ (if the process is centered).

Auto-correlative function $R_{xx}(\tau)$ is defined as

$$R_{xx}(\tau) = lim_{T\to\infty} \frac{1}{T} \int_0^T x(t).x(t+\tau)\,dt$$

Geometrical meaning of this function is shown on fig. 4.10, where is beside function course $x(t)$ plotted also its course shifted about time interval τ .

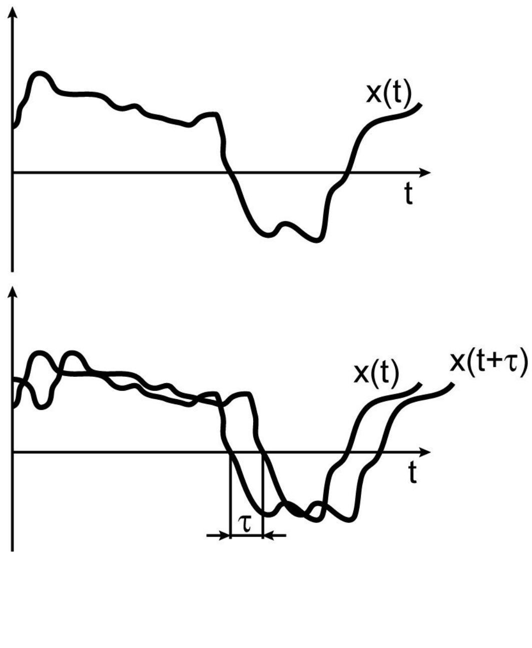

fig. 4.10

Both of these functions are slightly different. Measure of their difference (because of time shift τ) is area under the new curve that arise from the product *x(t).x(t + τ)* ; the height of rectangle with the same area answers the time correlative function.

In the similar way it must stand for centered processes auto-covariance function

$$K_{xx}(\tau) = lim_{T\to\infty} \frac{1}{T}\int_0^T \left[x(t)-\overline{x}\right].\left[x(t+\tau)-\overline{x}\right]dt = R_{xx}(\tau)-\overline{x}^2$$

Process's deviations from its mean value are called fluctuations.

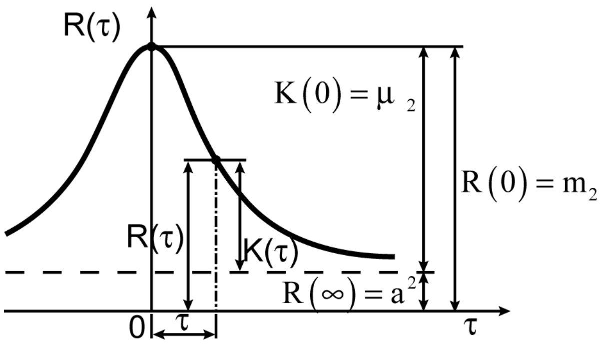

fig. 4.11

There are another qualities of the correlative function at the fig. 4.11:

$$lim_{\tau\to 0} R_{xx}(\tau) = R_{xx}(0) = m_2\left[x(t)\right]$$

$$lim_{\tau\to\infty} R_{xx}(\tau) = R(\infty) = \overline{x}^2$$

$$R(\tau) = R(-\tau)$$

$$R(0) > R(\tau)$$

Here can be seen an evident decline $R_{xx}(\tau)$ to the square mean value and fall $K_{xx}(\tau)$ onto zero for $\tau \to \infty$. In practice, there is able to speak about no correlativity of the stochastic processes from two cuts if the time difference τ is sufficiently big. This period is then called correlative interval (fig. 4.12).

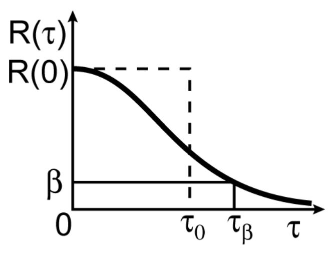

fig. 4.12

This one is defined as a:

a) Time τ_β, where the value of the correlative function falls under the chosen value of β, or

b) Length τ_0 of rectangle with the high *R(0)*, where its area is equal to area under the curve $R(\tau)$ for $\tau \to \infty$.

$$\tau_0 = \frac{1}{R_{xx}(0)} \int_0^\infty R_{xx}(\tau) d\tau$$

If it is added to stochastic function *x(t)* the deterministic function $\xi(t)$, we receive the following stochastic function

$$y(t) = x(t) + \xi(t)$$

For auto-covariance function then must stand

$$K_{yy}(\tau) = K_{xx}(\tau)$$

If the stationary process has a periodic part, it will be seen in a characteristic way on the course of auto-correlative function (fig. 4.13); then it is possible to fix the period of the periodic part.

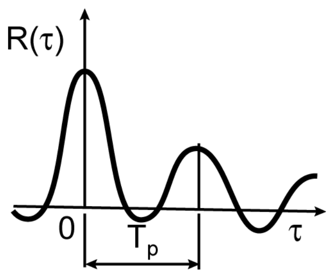

fig. 4.13

For the mutual comparison of the stationary processes is often used a standardized auto-correlative function (auto-correlative factor)

$$\rho_{xx}(\tau) = \frac{R_{xx}(\tau)}{m_2[x(t)]} = \frac{R_{xx}(\tau)}{R_{xx}(0)}$$

for it must hold true

$$\rho_{xx}(0) = 1 \qquad \rho_{xx}(-\tau) = \rho_{xx}(\tau) \qquad \rho_{xx}(\tau) \le 1$$

The examples of these functions of the deterministic processes are shown at fig. 4.14.

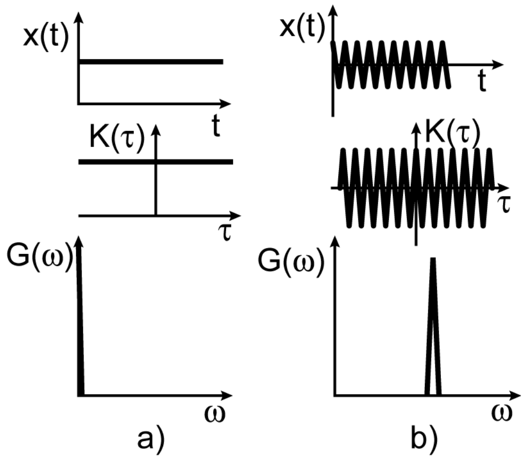

fig. 4.14

Here are:
a) Constant function,
b) Harmonic process.

There are shown the examples of the stationary stochastic process at the fig. 4.15.

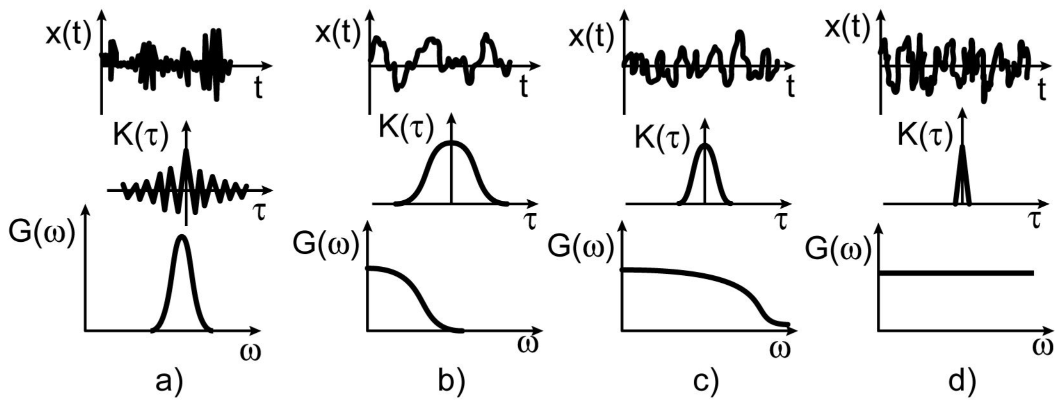

fig. 4.15

a) Narrow-zoned process,
b) Stochastic process with the frequency limitation of mean size,
c) Broad-zoned process,
d) Theoretical „white noise"

In conclusion it can be said that the technical meaning of defining of the correlative function is:
1. It informed us about frequency's part of the process, also the course of the power spectral density is giving a view;
2. Standardized auto-correlative (or covariance) function makes possible to compare two or more stochastic processes with respect to the type of correlative function;
3. Auto-correlative function also makes possible to analyze the stochastic processes containing periodical parts.

4.5. Characteristics in the frequency area

4.5.1. Power spectral density (PSD)

This characteristic describes a stochastic process in the frequency area. For two of them is then mutual PSD.

4.5.1.1. Physical image of PSD

We can assume ergodical centered process in the interval (0, T). Its covariance function $K_{xx}(\tau)$ is even function with the period $T = 2\pi/\omega$. In the interval (0, T) is possible to expand it to the Fourier's progression

$$K_{xx}(\tau) = \sum_{k=0}^{\infty} D_k . cos(\omega_k \tau)$$

Where (for $k = 0, 1, 2$ etc.)

$$\omega_k = k.\omega = k.\frac{2\pi}{T}$$

$$D_k = \frac{2}{T}\int_0^T K_{xx}(\tau).cos(\omega_k \tau)\, d\tau$$

Or also (for $k = 0, \pm 1, \pm 2$ etc.)

$$D_k = \frac{1}{T}\int_{-T}^{T} K_{xx}(\tau).cos(\omega_k \tau)\, d\tau$$

This term we will use in follow.

For $\tau = 0$ is

$$K_{xx}(\tau) = K_{xx}(0) = \mu_2\left[x(t)\right] = \sum_{k=0}^{\infty} D_k$$

D_k is the variance caused by frequency ω_k.

The distance between particular frequencies is

$$\Delta\omega = \omega_k - \omega_{k-1} = \frac{2\pi}{T}$$

The higher will be T the lower will be $\Delta\omega$ and also the softer will be the resolution of the general variance of the process owing to the frequencies.

We can create the quotients (fig. 4.16)

$$\frac{D_k}{\Delta\omega} = S_T^+(\omega_k)$$

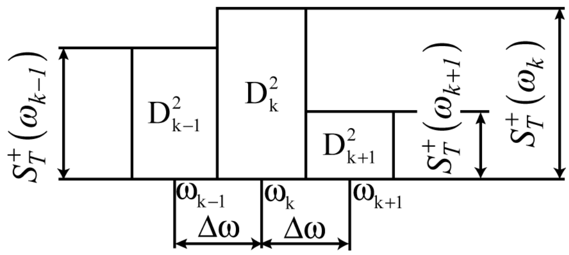

fig. 4.16

Then

$$D_k = \frac{1}{T}\int_{-T}^{T} K_{xx}(\tau).cos(\omega_k\tau)d\tau = S_T^+(\omega_k).\Delta\omega$$

And from that

$$S_T^+(\omega_k) = \frac{1}{2\pi}\int_{-T}^{T} K_{xx}(\tau).cos(\omega_k\tau)d\tau = \frac{1}{\pi}\int_{0}^{T} K_{xx}(\tau).cos(\omega_k\tau)d\tau = \frac{1}{2}G_T^+(\omega_k)$$

Is then

$$G_T^+(\omega_k) = \frac{2}{\pi}\int_{0}^{T} K_{xx}(\tau).cos(\omega_k\tau)d\tau$$

$G^+_T(\omega_k)$ is a function of PSD for $k = 1, 2$, etc.

For particular <u>*k*</u> is possible to defined the values of the function $G^+_T(\omega_k)$ and to them relevant variances. This particular variances D_k are caused by angle frequencies that lie in the interval $\omega_k - \Delta\omega/2$, $\omega_k + \Delta\omega/2$.

We can also considered that for the variance of the harmonic process (with amplitude *A*) must hold true

$$\mu_2[x(t)] = \frac{A^2}{2} \qquad\qquad A = \sqrt{2.\mu_2[x(t)]}$$

and also

$$A_k = \sqrt{2D_k} = \sqrt{2G_T^+(\omega_k).\Delta\omega}$$

What is the amplitude of harmonic process $A_k.sin(\omega_k t + \psi_k)$ that can be allocated to the particular frequency interval where the variances of harmonic process and D_k, are same. The value of the amplitude A_k characterized how significantly is in the stochastic process contained the particular frequency interval $\omega_k - \Delta\omega/2$, $\omega_k + \Delta\omega/2$.

For rising $T \to \infty$ will be $G^{+}_{T}(\omega_K) \to G^{+}(\omega)$, so as one sided PSD will be

$$G^{+}_{xx}(\omega) = \frac{2}{\pi}\int_{0}^{\infty} K_{xx}(\tau).cos(\omega\tau)\,d\tau$$

Similarly for $T \to \infty$ we would get also two-sided PSD

$$S^{+}_{xx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K_{xx}(\tau).cos(\omega\tau)\,d\tau = \frac{1}{\pi}\int_{0}^{\infty} K_{xx}(\tau).cos(\omega\tau)\,d\tau = \frac{1}{2}G^{+}_{xx}(\omega)$$

The dimension of the PSD is [U^2 / rad].

Similar procedure – substitution

$$D_k = S^{+}_{T}(\omega_k).\Delta\omega$$

is received

$$K_{xx}(\tau) = \sum_{k=0}^{\infty} D_k.cos(\omega_k\tau) = \sum_{k=0}^{\infty} S^{+}_{T}(\omega_k).\Delta\omega.cos(\omega_k\tau)$$

After getting an integral

$$K_{xx}(\tau) = \int_{-\infty}^{\infty} S^{+}_{xx}(\omega).cos(\omega\tau)\,d\omega = 2\int_{0}^{\infty} S^{+}_{xx}(\omega).cos(\omega\tau)\,d\omega = \int_{0}^{\infty} G^{+}_{xx}(\omega).cos(\omega\tau)\,d\omega$$

In technical applications is mentioned the frequency $\underline{f}$ than the circular frequency $\underline{\omega}$. And therefore for $\tau = 0$ must stand

$$K_{xx}(0) = rozptyl = \int_{0}^{\infty} G^{+}_{xx}(\omega)\,d\omega = \int_{0}^{\infty} G_{xx}(f)\,df = plocha$$

And from that with deliberation $d\omega = 2\pi df$

$$G_{xx}(f) = 2\pi.G^{+}_{xx}(\omega)$$

And also

$$G_{xx}(f) = 4\int_{0}^{\infty} K_{xx}(\tau).cos(2\pi f\tau)\,d\tau = 2S_{xx}(f)$$

The dimension of the PSD is then [U^2 / Hz].

PSD makes possible to get the power in the particular frequency zone f_1, f_2:

$$P(f_1, f_2) = \int_{f_1}^{f_2} G_{xx}(f)\,df$$

Also must hold true

$$K_{xx}(\tau)=\int_{0}^{\infty}G_{xx}(f).cos(2\pi f\tau)df=2\int_{0}^{\infty}S_{xx}(f).cos(2\pi f\tau)df$$

For no-centered process it is possible to replace $K_{xx}(\tau)$ by auto-correlative function $R_{xx}(\tau)$, so for example

$$G_{xx}(f)=4\int_{0}^{\infty}R_{xx}(\tau).cos(2\pi f\tau)d\tau=2S_{xx}(f)$$

$$R_{xx}(\tau)=\int_{0}^{\infty}G_{xx}(f).cos(2\pi f\tau)df=\int_{-\infty}^{\infty}S_{xx}(f).cos(2\pi f\tau)df$$

For $f = 0$

$$S_{xx}(f)=\int_{-\infty}^{\infty}R_{xx}(\tau)d\tau$$

For $\tau = 0$

$$R_{xx}(0)=m_2\left[x(t)\right]=\int_{-\infty}^{\infty}S_{xx}(f)df=\int_{0}^{\infty}G_{xx}(f)df$$

Mentioned connections are graphically shown at fig. 4.17.

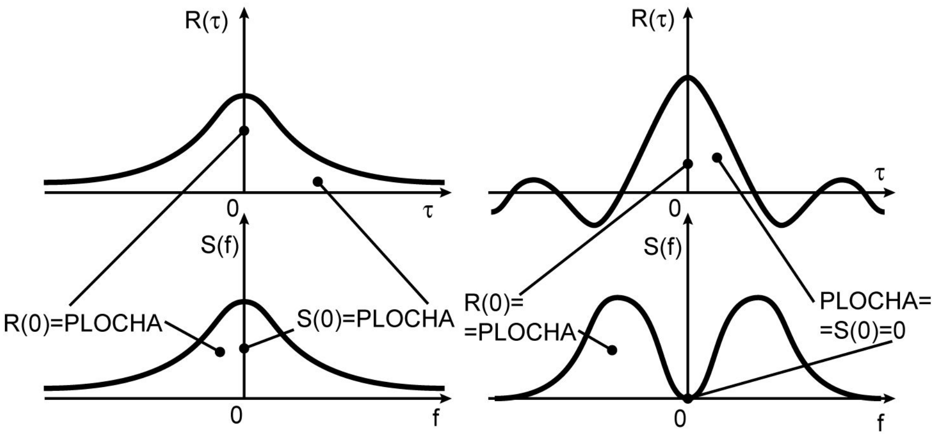

fig. 4.17

For mutual comparison of the functions of PSD is often used standardized function of the PSD

$$S_{xx}^{nor}(f)=\frac{S_{xx}(f)}{R_{xx}(0)}$$

$$G_{xx}^{nor}\left(f\right)=\frac{G_{xx}\left(f\right)}{R_{xx}\left(0\right)}$$

4.5.1.2. PSD and period-gram

If the time of measurement *T* is known, then it is possible to solve for every particular realization the ***frequency spectrum***

$$X_T\left(f\right)=\int_0^T x\left(t\right).e^{-j2\pi ft}dt=\int_{-\infty}^{\infty} x_T\left(t\right).e^{-j2\pi ft}dt$$

Function *Z(f)*

$$Z\left(f\right)=\frac{1}{T}\left|X_T\left(f\right)\right|^2$$

Is called ***period-gram***.

Mean value of these files' functions gives for T $\rightarrow\infty$ PSD of the stochastic process

$$S_{xx}{}'\left(f\right)=lim_{T\rightarrow\infty}E\left\{\frac{1}{T}\left|X_T\left(f\right)\right|^2\right\}$$

With the help of one realization is able to defined PSD

$$S_{xx}\left(f\right)=lim_{\Delta f\rightarrow 0}\frac{1}{\Delta f}lim_{T\rightarrow\infty}\frac{1}{T}\int_{f-\frac{\Delta f}{2}}^{f+\frac{\Delta f}{2}}\left|X_T\left(f\right)\right|^2 df$$

While $\Delta f\rightarrow 0$ and $T\rightarrow\infty$; from here

$$S_{xx}\left(f\right)=lim_{T\rightarrow\infty}\frac{1}{T}\left|X_T\left(f\right)\right|^2$$

4.5.1.3. PSD and Fourier's transformation Rxx (τ)

Beginning signal = ***original***.

PSD of the stochastic process *x(t)* is the Fourier's transformation its auto-correlative function

$$S_{xx}\left(f\right)=F\left\{R_{xx}\left(\tau\right)\right\}=\int_{-\infty}^{\infty}R_{xx}\left(\tau\right).exp\left(-j2\pi f\tau\right)d\tau$$

Since $R_{xx}(\tau)$ is real even function

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau).cos(2\pi f \tau) d\tau = 2\int_{0}^{\infty} R_{xx}(\tau).cos(2\pi f \tau) d\tau$$

And also

$$G_{xx}(f) = 2S_{xx}(f) = 4\int_{0}^{\infty} R_{xx}(\tau).cos(2\pi f \tau) d\tau$$

In the similar way it is able to get the correlative function by back FT of PSD

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{xx}(f).cos(2\pi f \tau) df = 2\int_{0}^{\infty} S_{xx}(f).cos(2\pi f \tau) df = \int_{0}^{\infty} G_{xx}(f).cos(2\pi f \tau) df$$

N. Wiener and A.J. Chinčin deduced these relations.

4.5.1.4. Different way of the spectrum presentation

Most often is the frequency spectrum presented with the help of **one-sided PSD** $G_{xx}(f)$ with the physical dimension [U^2 / Hz] where U is the dimension of an investigated quantity. Therefore for centered stochastic process must stand

$$G_{xx}(f).\Delta f = \sigma^2 = \frac{A^2}{2}$$

It is also possible to determine the equivalent amplitude of the harmonic process

$$A = \sqrt{2G_{xx}(f).\Delta f}$$

Where Δf is the difference of two neighboring parts of the spectrum; It answers to the resolution possibilities of that analyze:

$$\Delta f = \frac{f_{vz}}{N} = \frac{1}{N.T_{vz}} = \frac{1}{T}$$

Here is f_{vz} sampling frequency ($f_{vz} = 1 / T_{vz}$), N is the number of samples, T is the length of evaluated section. Maximum possible investigated frequency is then $f_{vz} / 2$.

For the spectrum with significant insulated parts is better way to use right **power spectrum**, where its components have the units of power [U^2]. Power spectrum is often received by multiplication of parts of PSD with the residual of frequencies of spectrum Δf.

Another possibility is presentation of the spectrum with effective values of its parts; particular parts that have dimension [U], are achieved by extraction of parts of power spectrum.

At the transitional processes is more suitable to give **energy spectral density**. Its components are rising by multiplication of parts of PSD with the time of record. They have the dimension [U^2.s/Hz].

4.5.1.5. Cepstrum

It is very useful to specify harmonic components in the frequency spectrum. Cepstrum (power auto-spectrum) $C_{xx}(\tau)$ is most often defined as a inverse Fourier transform of the logarithm of the PSD:

$$C_{xx}(\tau) = F^{-1}\left\{log\, S_{xx}(f)\right\}$$

The file of the harmonic components is then set with one maximum value.

For quantities definition have been used specific terminology which is based on the labeling of the particular quantity using an anagram:

Spectrum	→ cepstrum	harmonic	→ rahmonic
Frequency	→ quefrency	filtration	→ liftration
Etc.			

4.6. Characteristics of the system with two ergodical stochastic processes

We will understand by that two stochastic processes that are in a particular relation – whether geometrical or physical. It can even be the processes, where is on the first sight no evident relation.

We can presume hereafter system of two stationary ergodical processes X(t) and Y(t).

4.6.1. Mutual correlative and covariance function

These functions are defined by: see (fig. 4.18)

$$R_{xy}(\tau) = lim_{T\to\infty} \frac{1}{T}\int_0^T x(t).y(t+\tau)\,dt$$

$$K_{xy}(\tau) = lim_{T\to\infty} \frac{1}{T}\int_0^T \left\{x(t) - m_1\left[x(t)\right]\right\}.\left\{y(t+\tau) - m_1\left[y(t+\tau)\right]\right\}dt$$

$$K_{xy}(\tau) = R_{xy}(\tau) - m_1\left[x(t)\right].m_1\left[y(t)\right]$$

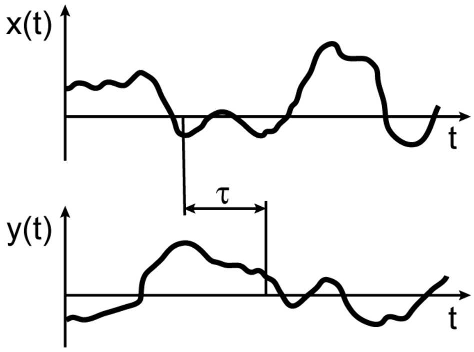

fig. 4.18

$R_{xy}(\tau)$ is a real function that can be positive as well as negative. In contrast to auto-correlative function, the mutual correlative function need not have its maximum at $\tau = 0$ and also need not be a positive function. There must hold true for it:

$$R_{xy}(-\tau) = R_{xy}(\tau) \qquad \left|R_{xy}(\tau)\right|^2 \le R_x(0).R_y(0)$$

$$\left|R_{xy}(\tau)\right| \le \frac{1}{2}\left[R_x(0) + R_y(0)\right]$$

If assumed stochastic processes are not correlated is their mutual covariance function zero for every shifts τ and mutual correlative function

$$R_{xy}(\tau) = \frac{m_1\left[x(t)\right]}{m_1\left[y(t)\right]}$$

For every τ.

If both of them are non-correlated, is the function $K_{xy}(\tau)$ at least for some τ zero and for some τ reaches maximum.

While containing *x, y* periodical part, has the mutual correlative function for bigger shift τ periodical course.

Mutual correlative function of the stochastic processes *x(t)* and *y(t)* is inverted Fourier's transformation their mutual PSD.

An example of mutual correlative function is shown at fig. 4.19.

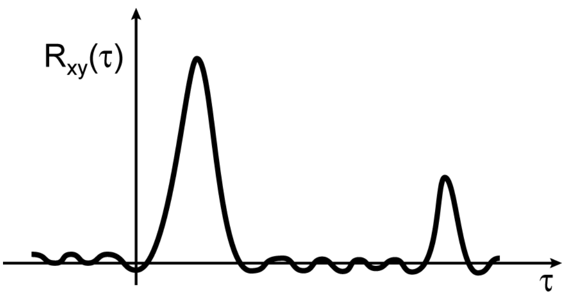

fig. 4.19

For comparison of these processes can be used standardized mutual covariance function

$$\rho_{xy}(\tau) = \frac{K_{xy}(\tau)}{\sqrt{K_{xx}(\tau).K_{yy}(\tau)}}$$

4.6.2. Mutual PSD

This function is given by Fourier transformation of mutual correlative function

$$S_{xy}(f) = \int_{-\infty}^{\infty} R_{xy}(\tau).exp(-j2\pi f\tau)d\tau$$

One-sided mutual PSD is

$$G_{xy}(f) = 2S_{xy}(f) \qquad \text{for } 0 \le f < \infty$$

$$G_{xy}(f) = 0 \qquad \text{for other } f$$

Because mutual correlative function is not even function, is mutual PSD is often complex quantity, it is form

$$G_{xy}(f) = C_{xy}(f) - jQ_{xy}(f)$$

Where
$C_{xy}(f)$ is coincidence spectrum; it is even function
$Q_{xy}(f)$ is quadrate spectrum; it is odd function

Mutual PSD is possible to defined in the form

$$G_{xy}(f) = \left|G_{xy}(f)\right|.exp\left[-j\Theta_{xy}(f)\right]$$

Where absolute size (modulus) is

$$\left|G_{xy}(f)\right| = \sqrt{C_{xy}^2(f) + Q_{xy}^2(f)}$$

And phase

$$\Theta_{xy}(f) = arctg\frac{Q_{xy}(f)}{C_{xy}(f)}$$

Example of these courses is at fig. 4.20.

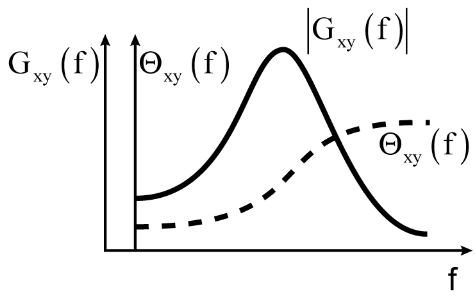

fig. 4.20

4.6.3. Coherent function

Is defined

$$\gamma_{xy}^2 = \frac{\left|G_{xy}(f)\right|^2}{G_{xx}(f).G_{yy}(f)}$$

If there is for any frequency $\gamma_{xy}^2\ (f) = 0$, then are the functions *x(t)* and *y(t)* no coherent, or no correlated. If the functions x(t) and y(t) are statistically independent, is then for all frequencies $\gamma_{xy}^2\ (f) = 0$. On the contrary, if for all frequencies is $\gamma_{xy}^2(f) = 1$, are these functions fully coherent. If the functions *x(t)* and *y(t)* are the functions of excitation and response, it regards with linear mechanical system with constant coefficient. In practical cases these limits are reached only seldom; for lifting the verdict we have to satisfy with condition that coherent function is getting near to zero or to one.

4.6.4. Frequency transmission

For linear system with constant coefficients with one input *x(t)* (that is realization of the stationary process) and with one output *y(t)* must be true

$$G_{yy}(f) = \left|H(f)\right|^2 .G_{xx}(f)$$

$$G_{xy}(f) = H(f).G_{xx}(f)$$

This makes possible to determine the amplitude and phase of frequency transmission.

4.7. Non-stationary stochastic processes

The results from the measurements of stochastic processes shown to us that mostly are no stationary. If this is not too expressive it is also possible to replace it by quasi-static process “per partes”.

No-stationary process is very often defined by time-dependence first and second density of probability as a function of time or also dependence of mean value and the variance on time.

Follow the example of the stochastic process

$$X(t) = \varphi(t) + Y(t)$$

Where the deterministic part is

$$\varphi(t) = A.cos(\omega t)$$

And *Y(t)* is centered stationary stochastic process with the density of probability $f_1(y)$. The density of probability of the process *X(t)* is $f_1(x,t)$ is shown at fig. 4.21.

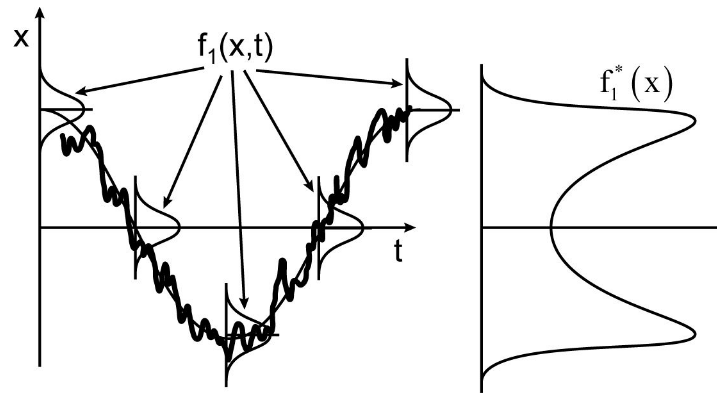

fig. 4.21

This expression that has deterministic and also stochastic parts and expressed fully the structure of the process. Less suitable is the second method where the process is presumed as a stationary and the density of probability $f_1^*(x)$ is expressed; by this way we indeed get rid of a no-stationary but in the form of density of probability $f_1^*(x)$ are also included information about deterministic part. It is not proper mainly for cases of simulation on an experimental device in the laboratory because the same course can have also a number pretty different process.

From mentioned is evident or visible the importance of the question of transmission the no-stationary process to the quasi-static process. (fig. 4.22).

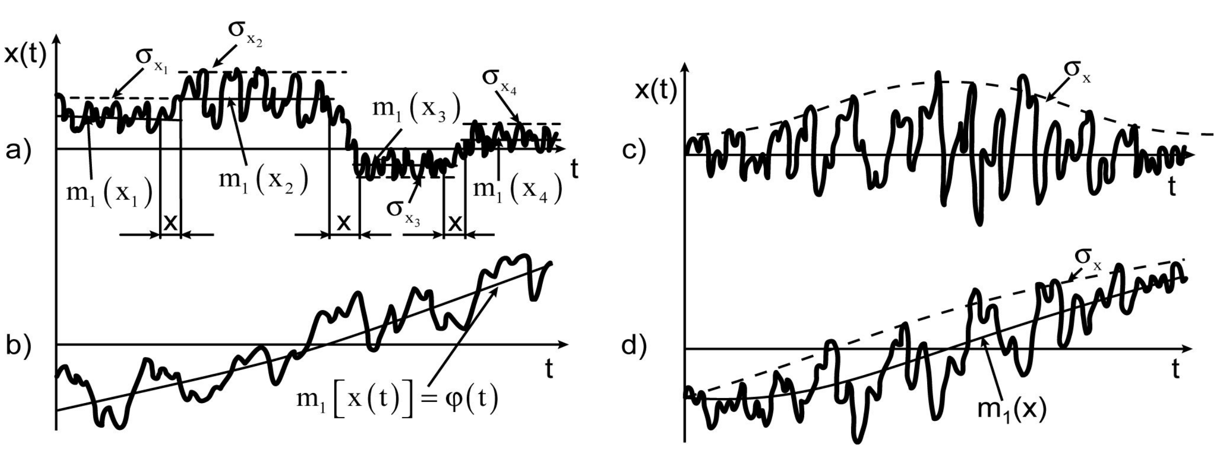

fig. 4.22

a) Process that is stationary "per partes". The parts of recording with the same statistical characteristic are added to bigger quasi-static files;
b) Additive no-stationary process has the following form

$$X(t) = Y(t) + \varphi(t)$$

Where *Y(t)* is a stationary stochastic process with the zero mean value and known correlative function, $\varphi(t)$ is deterministic function. It is possible to prove that

$$m_1\left[X(t)\right] = \varphi(t) \qquad K_{xx}(t_1, t_2) = K_{yy}(\tau)$$

Another type of additive process is in the form (fig. 4.23)

$$X(t) = Y(t) + Z(t)$$

Where *Y(t)*, *Z(t)* are stationary processes (while process *Y(t)* is centered).

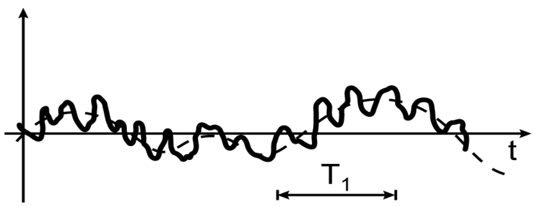

fig. 4.23

If the characteristic of the second order of the particular processes are significantly different from each other (if on a process *Z(t)* with slow changes in time is superposed essentially more faster changing process *Y(t)*), it is shown the essence of time-part length, where the stationary is judged: while long section, where the gentle process *Z(t)* is being averaged it is possible to assume the process *X(t)* as a stationary. For shorter sections – for example with the length T_1 – is indicated process *X(t)* as a no-stationary (no-stationary „in short"). For this type of process is valid

$$m_1\left[X(t)\right] = m_1\left[Y(t)\right] + m_1\left[Z(t)\right]$$

$$K_{xx}(\tau) = K_{yy}(\tau) + K_{zz}(\tau) + K_{yz}(\tau) + K_{zy}(\tau)$$

c) Multiplicative no-stationary process, may be like this type

$$X(t) = Y(t).\varphi(t)$$

Where *Y(t)* is stationary process with the zero mean value, $\varphi(t)$ is deterministic constant. The relation between covariance function $K_{xx}(\tau)$ and $K_{yy}(\tau)$ is basically complicated than in previous cases. While the processes are being that are often presented in practice, must hold true

$$K_{xx}(\tau,t)=\varphi(t).K_{yy}(\tau)$$

Well, it means that proportional distribution of the frequency parts is for the processes *X(t)* and *Y(t)* the same and the difference is only in their variance.

d) Additively multiplicative no-stationary process is rising with superposition of a multiplicative no-stationary process and a deterministic mean value. By this way it is possible to describe each and every loading process in practice.

Above-mentioned form of processing of any no-stationary processes are taking mainly widespread experience and knowing of physical basement of the problem and also the cause of the no-stationary process. It's of course needful to verify these results of data processing.

The second way of data processing does not assume any precarious characteristic of the process. It is based on using so called evolutional characteristics of no-stationary stochastic process. These ones contain implicitly time as a parameter. Practically it means that these char. Are not invariant and thus they are addicted to the beginning of the processing.

4.8. Literature

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5. ELECTRIC RESISTIVE STRAIN GAUGES

5.1. From the history of resistive strain gauges

„**Tensometer**‟ was originally marking for one particular construction of a mechanical extensometer from Huggenberger Company, which one was in Czech language gradually translated also for other extensometers. In English language is often used the term „strain-gauge‟ or „strain-gage‟, and in German language „Dehnungsmeßstreifen‟ – DMS.

Practical using of strain gauges could not be possible without two discoveries:

- Relation between stress, resistance and electrical current that was written by G.S. Ohm in 1827.
- Suitable wiring for measuring of the small resistive changes – Charles Wheatstone 1843.

  Physical principle, whereupon are based all strain gages – relation between resistance of line wire and its extension – was discovered already in 1856 by lord Kelvin (original name William Thomson, 1824 - 1905). This discover had not been a long time used.

  Production of strain gauges has rising during the Second World War and in coming times.

5.1.1. Strain gauges in ČSR

In the Czech Republic as a first typical using of resistive strain gauges was in stiffness measuring of the aircraft Me 262 Schwalbe done in 1948 in the Experimental aero institute in Prague.

The first Czech-Slovak strain gauge was produced by Štěpán Kobylka in company Aero in Prague - Holešovice. It was winded from a material Advance or HW 50 on the paper base. It had four loops, resistance 120 Ω and it was applied and fixed by using butyl-methyl-metakrylate glue. Later, their production was starting in the national company Mikrotechna.

5.1.2. HBM Company and strain gauges

In Germany was for the first time applied a strain gauge by Theis in 1941 in the laboratory of AEG Company. His strain gauge had a graphite resistive layer.

During the Second World War Karl Hottinger had been working in the experimental center Luftwaffe in Rechlinu, where he had been dealing with measuring of mechanical entities. Following his war experience was set up in 1950 Hottinger Messtechnik Company in Vogtareuth. He directed himself on production of inductive sensors of displacement and also measuring amplifiers with carrier frequency 5 kHz. In 1955 was his firm connected with Carl Schenck Maschinenfabrik GmbH in Darmstadt. At this time Schenck Company bought the license for production of strain gauges from the firm Baldwin-Lima-Hamilton Corp. (USA). This Company had been producing wire strain gauges with the flat gauge and also winded gauges so called „wrap around‟.

During its development strain gauges has received very high level of perfection and also nowadays its development has not been ended.

Widest using the strain gauges have been having in the branch of experimental analyses of stresses, forces, pressures and twisting moments. Of course they are able to measure also displacements, shifts and acceleration of an oscillation.

They have following **advantages**:

- Enable transferring of all measured data to long distance,
- Enable easily further working up of measured data,
- Make possible to measure static and also dynamic stresses to the high frequency. The maximum of possible measured frequency is as a rule given by characters of measured device and far from the gages,
- They have low weight and only by low rate influence the measure object,
- Enable measuring on the different curve or flat areas,
- Provide possibility of measuring for high and low temperatures,
- Fixing on the measured area is very simple and relatively fast.

5.2. Division of strain gages and their basic characteristics

I. Metal gages

Design of a metal gage is

- **Wired** (historically older, in nowadays only for special purposes),
- **Foil** (make possible different shape of the gages),
- **Layered** (mainly for pressure sensors).

a) Wired gages

- **Glued**
    - **With fundament**

This is oldest type of strain gages. Gage is created from a wire and fixed to fundament by glue. In experimental elasticity it is used for determination of uniaxial and multiaxial state of stress, residual stresses, detection of concentrators and gradients of stresses at level of elastic and plastic deformations. Special types make possible to measure during extreme temperatures from –270 to +950 ^{0}C, measuring during high long-time dynamic stress (1000 cycles while relative deformation being $\pm$ 4000 μm/m). By choice of gage's material is possible to create self-compensating gages for different types of material (most often α = 12 ; 23 ; 16 ; 9.10^{-6} $1/^0$C) with minimal length of an active gage 1 mm.

For wired gages is guaranteed small variance of the resistance, small variance of k-factor and also temperatures' factors. Their price is acceptable and for rich assortment of gages exists corresponding assortment of glues. Their disadvantage is limited attainable value of resistance, low value of k-factor.

- **Without filler**

(These gages are marked as „the gages with free grid“!)
Using: High temperature gages that are fixed with ceramic binder.

- **The gages with free grid (not glued)**

At this type of gages the resistive wires are fixed between the systems of holders. Active resistive piece is not glued and therefore here are not the problems with transmitting of deformation to whole area of gage. Another advantage is it that this type makes possible to use for higher temperature (till 310 ^{o}C). They have low hysteresis and also small shift of zero point. Disadvantage is low resonance frequency (units kHz a lower) and relatively long time needful for achieving thermal balance as well. Their production is very difficult due to mechanical design.

b) Foil gages

In nowadays these are the most used one from the type of metallic strain gages.

The measure grid is made from foil (minimum width is only 5 μm) and glued to the prepared area. Etching mostly creates a shape of grid. The basis or fundament is made from polyamide or from phenol layers or fibres.

For this type of gage is possible to create different arbitrary configuration of measured grid (minimum length 0,4 mm) and also can be used higher supply voltage.

Another step – during their production – is etching of all measured grids with their connecting and compensating grids from one foil („laminated technique“). As a one complex is that fixed to the measured object or area.

For construction of very small types the gluing of foil gages has however the cause of adverse values of some characteristics as a creep and hysteresis. These drawbacks however haven't following type of gages.

c) Layered

Preparation of layered gages:

On an elastic material is first piled up dielectric ceramic layer and then metallic layer. Then follow spreading of photosensitive mask, exposition by required picture of gages and removing no exposed parts of metallic layer.

By this method is achieved high value of resistance, arbitrary configuration of a measured grid, good transmission of the deformation of elastic material to own grid, long-time stability and also reproducibility.

This method is used only for construction of pressure sensors. Gages are created right on the backside of measured membrane. Used electronic is on the tablet of flat lines, which is elastic and it is fixed right to action unit. By this way wire connection (by soldering) fall off. Inside of sensor is used costumer's integrated circuit that can be adjusted by suitable software for different values of current or voltage outputs.

II. Semi conducting gauges

They are based on piezo-resistance phenomenon of some materials that was discovered in 1954 American physicist C.C. Smith.

The first semi conducting gages were made in 1959 in the USA. Main problem here was receiving for the silicon, that is fragile as a glass, immunity against tension deformation comparable with deformation of metallic material of analyzed construction.

Even if they are at theoretical level sufficiently elaborate, they didn't achieve the level and the degree of spreading of the metallic gages. Here are a quite number of problems with their sensitivity, behaving toward thermal changes.

Their perfections are:

- Higher sensitivity factor - almost 100 x bigger than for metallic gages,
- High fatigue life-time (suitable for dynamometer of fatigue devices),
- Excellent stability (in a broad interval of temperatures the hysteresis, drift and also creep is not rising),
- Small size,
- High resistance.

5.3. The types of resistive gages

Resistive gages are made in different types and sizes. Also existing are different types of placing of measured grid and different types of soldering spots. The differences are in position as well as in the number of measured grids (fig. 5.1).

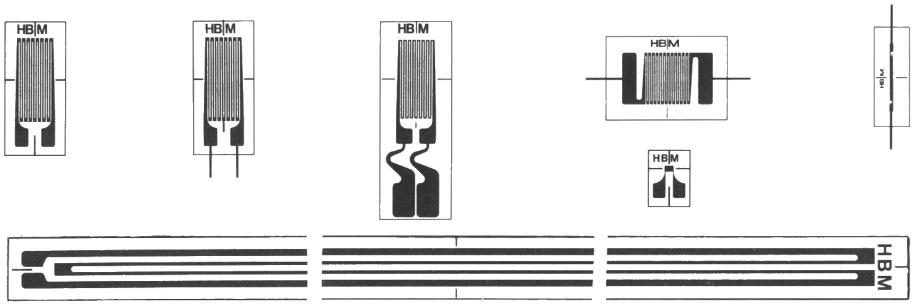


fig. 5.1

5.3.1. A length of the measured grid

These gages are currently produced with the length of measured platform 0.6mm to 150mm. The main criterion for choice of the length is homogeneity in the place of gage application.

The sensitivity of gage is independent on its length. Sensitivity of metallic gages depends on relative elongation. It means that that the size of gage hasn't any influence on its sensitivity. Nevertheless, gages with extremely small length of measured platform are used only where they are necessarily needful for instance if we investigate the stress's field at a notch. This way is better than cutting off any parts of gage's platform. In the case of platform's trimming happens that the transmitting path of the stress from measured place is broken. The strain gages are produced in order not to intervene the influence of overpasses among the layers to the active part of measured grid. That can happen if the platform is cut off.

5.3.2. Multiple strain gages

Multiple gages consist of many single measured grids on one common platform or fundament. Typical examples are gages' crosses, gages' rosettes and gages' chains. Their particular measured platforms are on one common basement placed onto exactly oriented directions and with exactly prescribed span.

5.3.2.1. Gauges' chains

Gauges' chains are combination of measured grids of the same type on the common platform in regular intervals. Chains with the rosettes or crosses often have 10 measured grids or 5 groups with 3 measured grids. At the end of all grids is placed one grids used as a compensating or completive grid. The main their use is for detecting of a gradient of stress. (fig. 5.2).

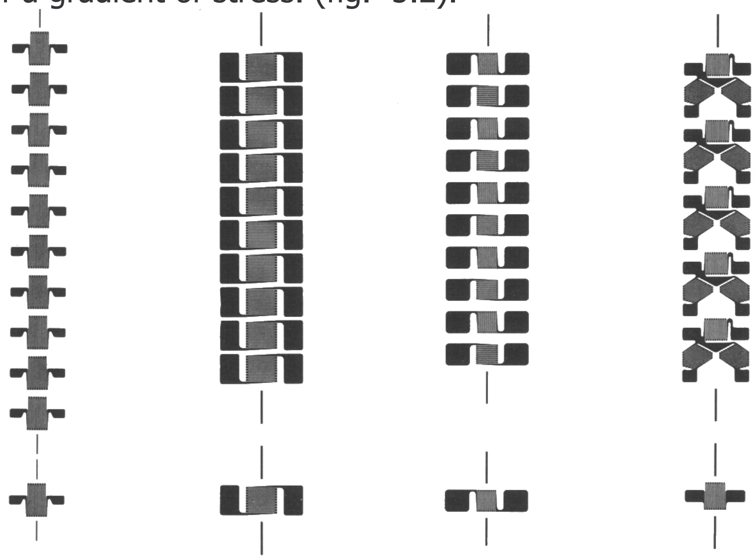


fig. 5.2

5.3.2.2. Gages' crosses and rosettes

At investigating of state of plane stress is needful deformation measuring in many directions. For measuring known direction of main stresses are used gages' crosses that have two independent winding below angling 90°. There are many types of that, the most known are types like these X, L, T and V, named according to the shape of crossing of measuring platform.

In the case that there aren't known the direction of main stresses for investigation of state of the plane stress, it is needful to measure the deformation at least in three independent directions. For that purpose is often used gages' rosettes. They are produced in two basic forms that are different from each other by angle displacement of single measuring basement or platform – 0°/45°/90° and 0°/60°/120°. Similarly as in gages' crosses are here produced many types of them that are different in placing of measuring grids on the platform.

In the special cases are used rosettes with four grids. The 4-th winding is simply arbitrary – it is used for comparison of results by Least Square Method and by this way improvement in accuracy of measurement. It is also recommended in cases with high possibility of damaging some of the grid or in places with difficult approaching. (fig. 5.3).

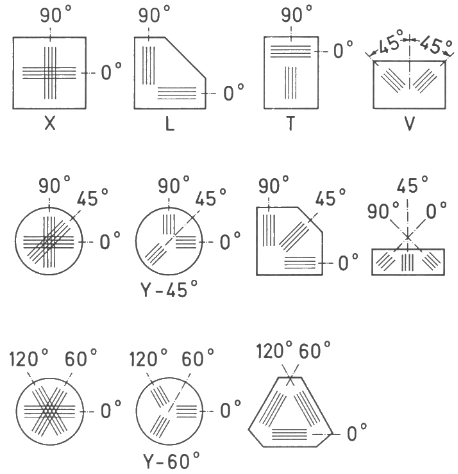


fig. 5.3

5.3.2.3. Rosettes for measuring of residual stresses

For measuring of residual stresses are nowadays for use two methods. Both of them are destructive and are based on principle of putting any material from the measured object. The older method „hole drilling method‟ is based on over division of stress's field and deformation causes due to drilling the hole in the centre of rosette. For this method are used rosettes with three windings with angle displacement 0°/45°/90° about the centre (fig. 5.4).

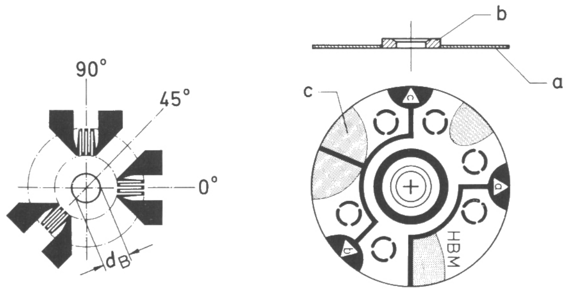


fig. 5.4

The second of them is „ring – core method‟, uses also the gage's rosette with three windings under the same angle as above, but the difference is in the places of drilling the material. Here are by ring-drill taken off the column of material round the circumference of gage. (fig. 5.5).

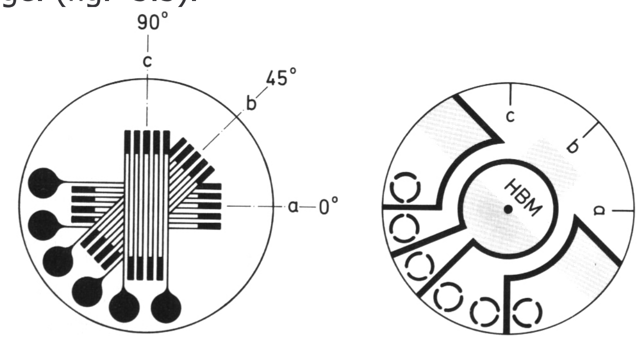


fig. 5.5

5.3.2.4. Welding gauges

They have a thin metallic platform where is cemented measuring grid. To the measured object is fixed by spot-welder whereby is limited using only for steels and tempered iron. They are stiff enough and therefore they are used on thick sides. (fig. 5.6).

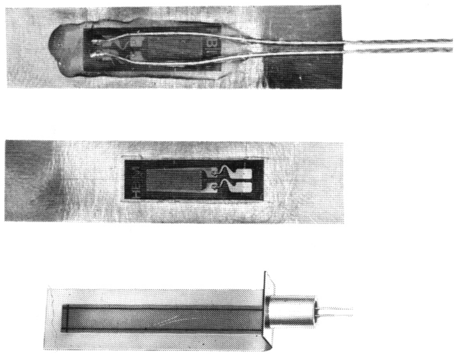


fig. 5.6

5.3.2.5. Gauges with free grid

This type of gages is often used for measuring at extremely high temperatures (till 1000°C) or low temperatures (-200°C). Measuring grid is fixed on the flat platform that is taking off during fixing. For the fixation is used special ceramic spray. Some of them are with thermocouple that makes possible to compensate the temperature. (fig. 5.7).

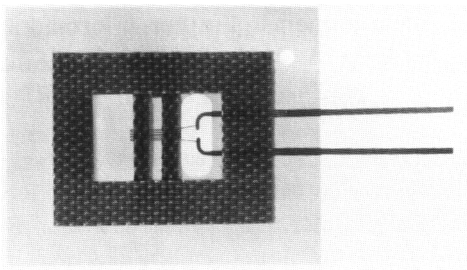


fig. 5.7

5.3.2.6. High-temperature gauges that are welded

This type of sensors is predicted for long-time or respectively lifetime measuring during favorable conditions. These gages are mounted on metallic foil and wires are done by shielded cable. They are supplied in ¼- or ½-bridge wiring. (fig. 5.8).

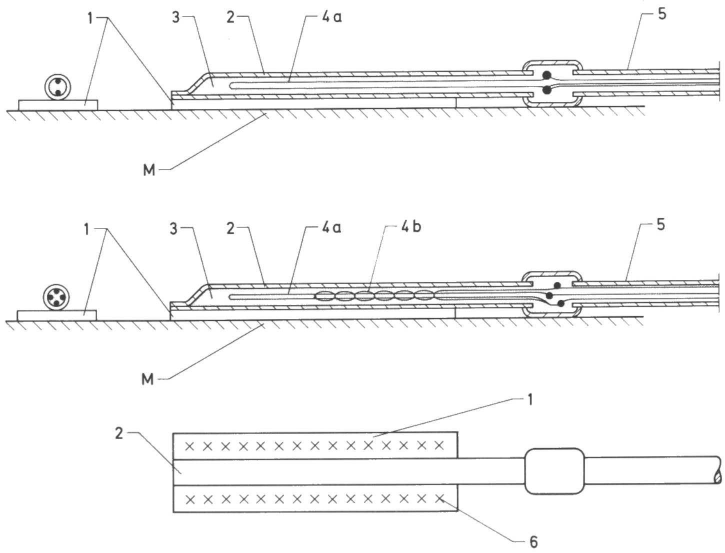


fig. 5.8

5.3.3. Electrical resistance

Gages are produced with different values of any particular resistance. Nowadays are mostly used gauges with rated resistance 120Ω. With reference to balancing of measuring bridge is necessary so as every engaged sensors have the same value of resistance or very slightly different. Common tolerance is 0,5% of rated value of gage's resistance. Gages in one package usually answer this requirement. Of course, by not proper application (for instance – bended gage) can arise a immense change of resistance.

5.3.4. Gage's factor of deformation

The function of strain gage is based on length changing of a metallic sensor that courses a change of its resistance *R*. This change, is given by relation

$$R = \rho \frac{L}{S},$$

Where ρ, L, S is specific resistance, length and cross sectional area of resistive wire.

For finite change ΔR of resistance R, is possible to deduce the relation:

$$\frac{\Delta R}{R} = k\varepsilon.$$

Where k is gage's factor of deformation (so called „k-factor")

K-factor is dimensionless proportional factor that it in itself contains not only the influence of measuring gage but also the whole configuration of the sensor. On that account producer does the measuring of k-factor on statistically significant numbers of gages and shows on every package the value of the k-factor with tolerance. For grids made of constantan alloy is this value about 2.

Dependence between resistance's change and strain is not quite linear. Real function is parabolic. But, till certain value of strain is this difference between real function and linear function very small and therefore it can be neglected. This value is depended on the used material for gage's production.

Another parameter that affects the value of k-factor is temperature. The producer executes the statistical measuring on gages at room temperature. Producer of course gives temperature coefficient whereby is possible to recount the value of k-factor from room temperature to the temperature of measured place. The dependency between change of k-factor and temperature is not also linear but the linear function is quite good compensation.

5.3.5. Cross sensitivity

On gage should be change of resistance and by this way the strain only in active length of the sensor and their rate should mean the k-factor. Sometimes can happen that a deformation is significant in cross direction against of active length. This is important for measuring. So, the cross sensitivity of a gauge is defined like this:

$$q = \frac{k_t}{k_l}$$

Where k_l is k-factor in the direction of active length and is defined

$$k_l = \frac{\Delta R / R_0}{\varepsilon_l}$$

And k_t is k-factor in the cross direction against active length which is defined

$$k_t = \frac{\Delta R / R_0}{\varepsilon_t}$$

For prevention of cross sensitivity is most often used thickness at the end of particular loop of measuring grid.

The second effect that is connected to cross strain is necking in the area of active length of grid affected by cross contraction of the material. This effect can cause only small negative change of resistance for positive strain. Gages have the cross sensitivity commonly lower than 0,008.

5.4. Gage's response on temperature change

5.4.1. Virtual deformation

If there is any changing of the temperature after mounting the sensor on the measured object, there will be change in the value of measured deformation as well. To that reality support three factors:

- Thermal coefficient of expansion α_S of measured material,
- Thermal coefficient of expansion α_G of grid's material,
- Thermal coefficient of el. resistance β_G of grid's material.

Gauge's resistive response developed by the temperature's change can be determined as a sum of resistive change as a cause of different expansions grids and measured material, and resistive change of grid as a cause of warming-up:

$$\frac{\Delta R}{R} = k\left(\alpha_S - \alpha_G\right).\Delta T + \beta_G.\Delta T = k.\varepsilon_z$$

Virtual deformation ε_z developed by temperature's change is then

$$\varepsilon_z = \left[\left(\alpha_S - \alpha_G\right) + \frac{\beta_G}{k}\right].\Delta T = \left(\frac{\beta_G}{k} - \alpha_G\right).\Delta T + \alpha_S.\Delta T$$

Where k is k-factor
ΔT is change of temperature

Remark.
In this context is possible to determine idea of „thermal coefficient of measured place" α_M

$$\alpha_M = \left(\alpha_S - \alpha_G\right) + \frac{\beta_G}{k}$$

Then is

$$\varepsilon_z = \alpha_M.\Delta T$$

Dependence of virtual deformation on temperature states the producers of gages in the form of multinomial, which is suitable for correction of its influence.

An error that can be given by virtual deformation may be pretty big if the temperature of environment is quite different from referential temperature (that is in common room temperature).

The example of statement of virtual deformation on the same gauge that is fixed a) on aluminum, b) on steel and c) on silicon is shown at fig. 5.9.

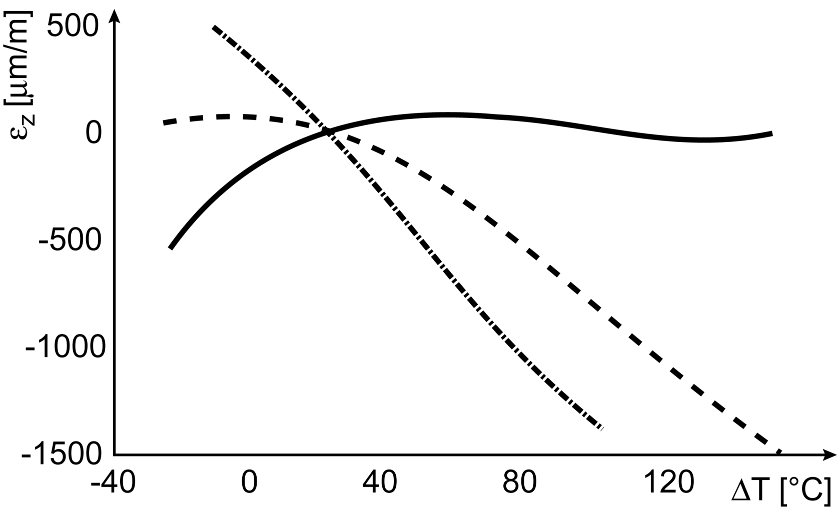


fig. 5.9

Manifestation of virtual deformation is reversible.

5.4.2. Thermally self-compensated gauges

There do exist such the ways of production and manufacturing of measuring grid, where is possible to achieve mineralization of virtual deformation in the certain thermal interval. First possibility is to change the thermal coefficient of el. resistance β_G of grid's material. This one can be influenced for example by modification of chemical constitution of grid's material, by thermal and mechanical processing. Size of possible interference provides fig. 5.10

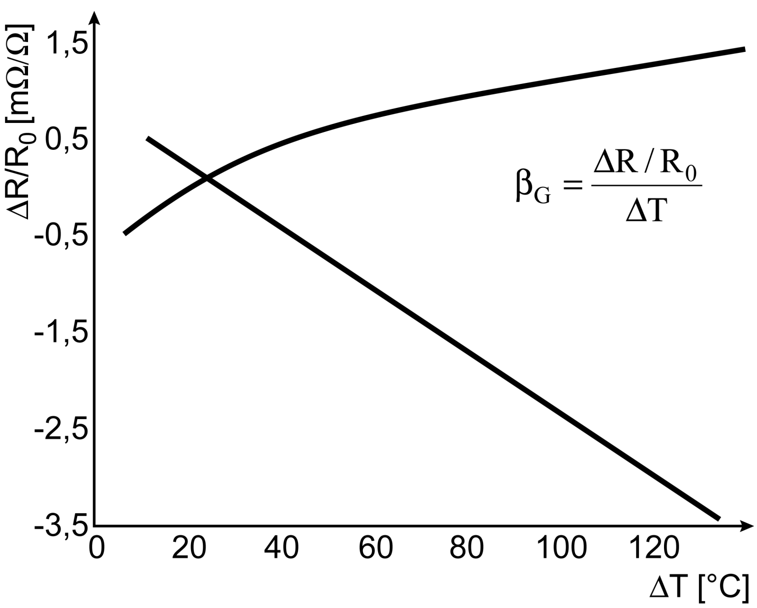


fig. 5.10

There is possible to attain positive as well as negative values of β_G's change.

An effort is to achieve such a value of β_G so that this condition will be fulfilled

$$\beta_G = (\alpha_G - \alpha_S).k$$

The values of particular members aren't linear function of temperature and therefore it is not possible to attain totally self-compensating gauge.

An optimal conformity thermally self-compensated gage to the thermal coefficient of expansion of object's material is possible only in the case of gaga application on the flat surface. There are some errors of that optimum for gages fixed on the curved areas.

5.4.3. Thermal drift

Thermal drift rises mainly by micro structural changes and oxidation or corrosion of measuring grid. Another possible cause is for instance stress relaxation that is situated in measuring grid or in glue as a cause of influence by temperature changes. These processes are addicted to time and to temperature. Thermal drift isn't reversible change and there is no possibility to attain the original zero point.

The important factors influenced thermal drifts are constitution of grid's material and thermal as well as mechanical processing of grid. It is known that cold-drawn or sheeting materials for grids have had more important drift which is apparent at low temperature about 100°C. An influence has also the history of mechanical processing of gage's grid.

The errors caused by thermal drift can be during measuring excluded by suitable bridge wiring of gages.

5.5. Boundary condition during static tension

Using of common gages is limited to interval $\varepsilon = \pm 3000$ μm/m. In some case is needed to measure above that limit. Maximum measured strain depends on gage's construction and its material. There do exist special gauges that make possible to measure strain till 20 cm/m.

Measuring grid of gage in plastic area comes down its original characteristic. It is not possible in this area does repeatable measuring or in very short limits. It is not proper to do the measuring on materials like rubber etc.

During measuring of big strain happen rising of nonlinearity and this is on the side of gauge and also on the side of Wheatston's bridge. These nonlinearities are significant and there is no possibility to neglect them.

At micro structural changes in the material of measuring grid under the influence of plastic deformation happen the change of thermal coefficient of resistance. That's why there can't be produced these gages for big deformation as a self-compensated. Gages with thermal compensation come down this behavior at plastic deformation.

5.6. Boundary condition during dynamic tension

There is no problem to use the strain gauges for dynamic measuring. Because they have minimal weight there don't happen influence of measured place. During dynamic measuring is necessary to consider two limiting factors:

- Fatigue's and fracture's characteristic of sensor
- Upper frequency, where there can be measuring reliable.

5.7. Fatigue of gauge

In the case of gage's tension by constant amplitude can happen that in amplitude indication of the loading force will be inequality or shape distortion. This problem rises in measuring grid and fixing contacts.

At gages with metallic grid can be damage displayed by two ways:

- Resistance addition as a cause of amplitude and change of tension. That can happen owing to the drift of dynamics zero.
- By rising damage of material, microscopic cracks on the edge of grains in material of measuring grid.

From the number of experiments followed on that the gages with longer grid have slightly better fatigue characteristics than the gauges with smaller grid.

A producer of gages sets the function of zero's shift in relation with an amplitude of deformation for alternate cycle on the number of cycles. (fig. 5.11)

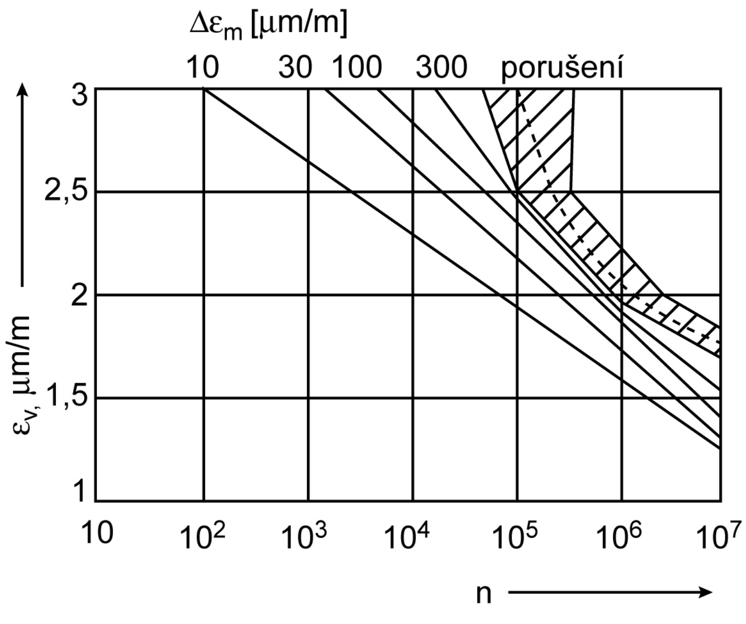


fig. 5.11

5.8. Marginal frequency

In the case of tension that can give rise of longitudinal waves in the material as are for example impressions may attain the state where strain has so low wave length that gage by its integration ability can averaged rising peaks and shows to us only the mean value of existing strain. For proper measuring must be the rate of active length of gage and wave's length of walking longitudinal wave as smallest as it can be. Only in this way we are able to measure the peak's value. For measuring of impressions and quite fast dynamics processes is recommended to use the gages with maximum length of grid 3-6mm. (fig. 1.12)

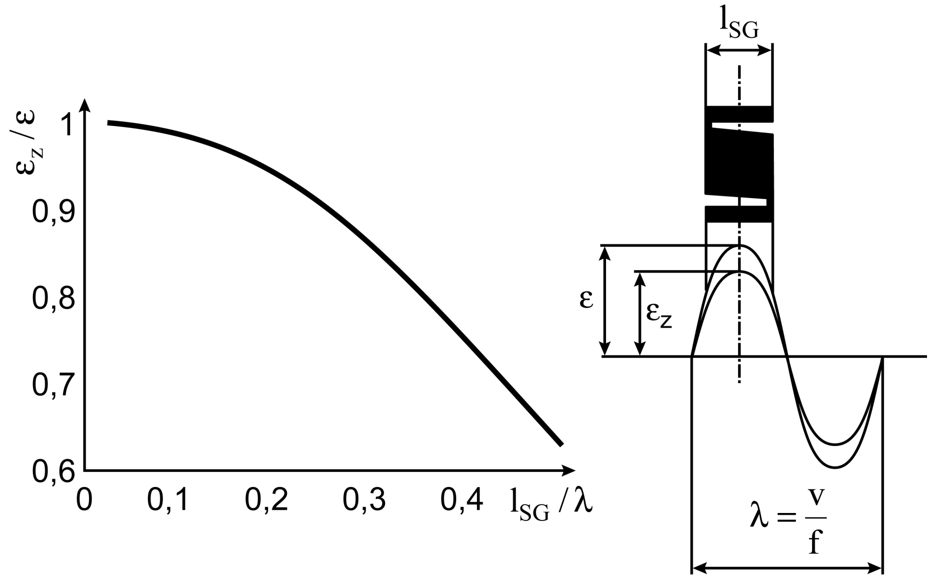


fig. 5.12

5.9. Supply voltage

If through the gage with the resistance 120Ω is flowing for instance the voltage 5V, then the current loading about 20 mA. Then also quite small change of voltage can cause big current loading. This can in following cause warming of measuring grid and cover layer, bending of platform, cause hysteresis, creep and instability of zero point.

On that account the producer gives maximum possible voltage loading for particular gauges. In the case that we measure at condition where these ones influenced the gage (for example temperature of environment) is necessary walking voltage reduced.

5.10. Creep

The effect of creep is rising during the installation of grid on the measured place when in case of constant static loading after a while the measured quantity is getting down (fig. 5.13). Creep causes material properties of particular layers by these are the strain transferring to the measuring grid. On the quantity of creep the used glue has also certain effect. The methods how to compensate this creep are two. We can use gages with special construction or use elastic „after" effect, which has the same function as a creep but it is exactly opposite.

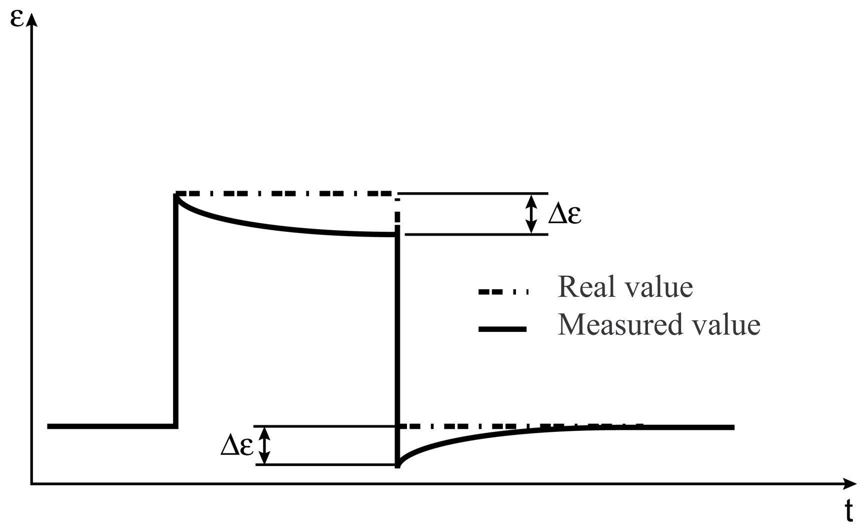


fig. 5.13

5.11. Hysteresis

Hysteresis at sensors is the difference in value of measured resistance change while rising and falling of strain. This factor depends also on the quite a number of parameters like measured place, fixing, platform are. Experiments show that the hysteresis is stabilizing on constant value after a number of loading cycles. While proper gluing of gage is usually hysteresis in limits (0,25 –0,5%) of measured deformation (fig. 5.14).

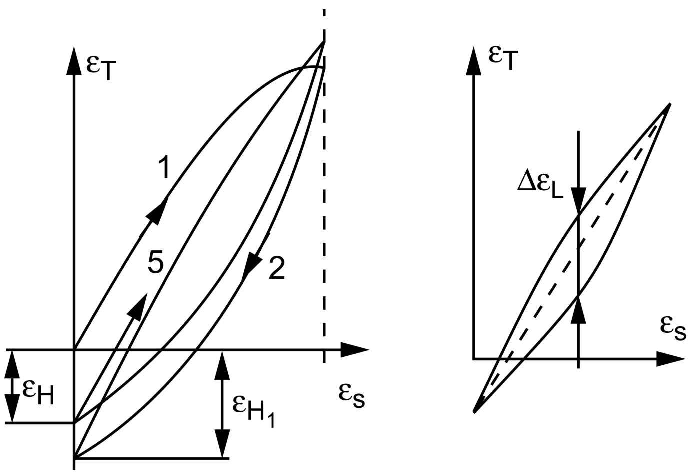


fig. 5.14

5.12. The influence of service conditions on the gages' properties

The service conditions contain not only the effect on sensor but also the effect on measured place. The accuracy is influenced by many factors.

5.12.1. Humidity

As well as the temperature and the humidity can cause any problems during the measuring.

Humidity affected the measured place by penetrating through the isolation, decreasing an insulating resistance between the grid and measured place. Changes of humidity can cause bending, recover or change of glue's characteristics. High humidity can also cause total ungluing or gage's corrosion. It is necessary to use suitable tool for covering and strictly hold the conditions for their application.

5.12.2. Hydrostatic pressure

It is known that the gages can stand pressure 1000MPa without any damage. Under pressure loading rising small change of resistance as a function of pressure, which is displayed as an error of measuring. The most important factor here is the quality of gluing.

Used glue mustn't contain any dissolvent. It is necessary to use such glue that will create very thin equal layer under platform with measuring grid. The glue also mustn't have necessarily any bubbles that can give rise of hysteresis, change of zero point or damage of grid.

5.12.3. Radioactivity

There do exist several types of nuclear radiation – radiation α, β, γ and neutron's radiation.

By incidence of that radiation happen that some organic parts of gages and glues are badly influenced. There is needful using of some special gauges, which are proper made for conditions like that.

5.12.4. Effect of magnetic field

By incidence of magnetic field during the measurement happen:

- Magneto-striction of measured object (change of geom. size causes virtual deformation),
- Magneto-striction of grid's material,
- Magneto-resistance of grid's material (magnetic field causes the change of el. Characteristics of used material),
- Rising of el. Voltage in the grid and connections by changing of magnetic field.

If the ferro-magnetic material is put to the magnetic field incidence due to magnet-striction happen geometrical changes of that material that are transmitted to the applied gage.

For prevention to these unfavorable incidences is used for instance shielding of measuring circuit or there are used no inductive sensors where the half of measuring grid is winded on the opposite direction.

5.13. Installation of strain gages

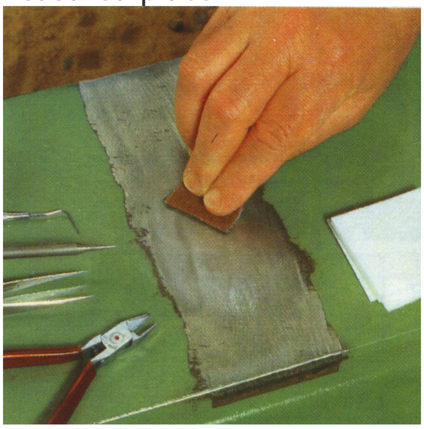
5.13.1. Preparation of the measured surface

5.13.1.1. Preparation of a metallic surfaces

The condition for proper connection is roughened sticky surface.

Precleaning

Rust, flakes, paints and similar rough dirties is necessary to put it away from sufficient extent round the measured place.

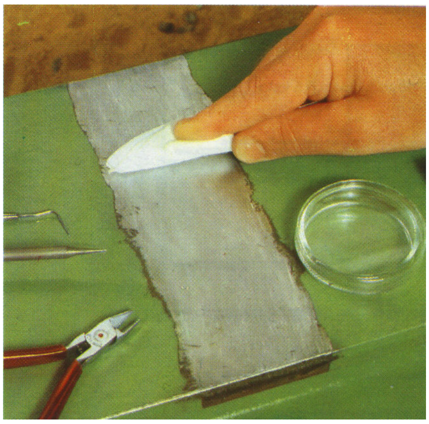


Alignment

Some holes and nodes are necessary to grind down or to put it away in another way.

Degreasing

Choice of proper degreasing tools and solvents is depended on the way of pollution. Suitable are strong solvents as a methylethylketon, acetone. Waxes and similar substance is possible to dissolve in toluene.



Roughage

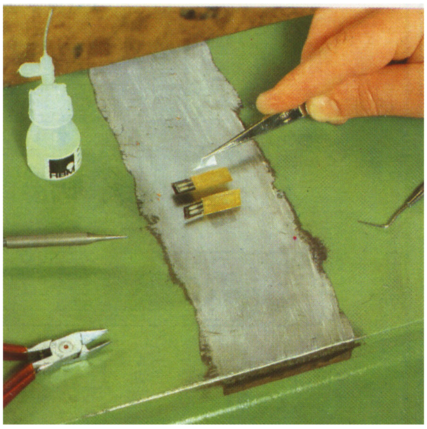
If the surface is slightly roughed so the glue has the best adhesion. By sanding or etching can be attained that state. Where there is no possibility to use these methods we can also use emery paper with granularity about 180 -300.

Final purification

Dust and dirties that arisen while roughage must be totally put away. For that we can use solvent and unwoven stuff with tweezers. Pertinent fibres from unwoven stuff may be removed by silky paper. To touch this place by fingers is strictly prohibited.

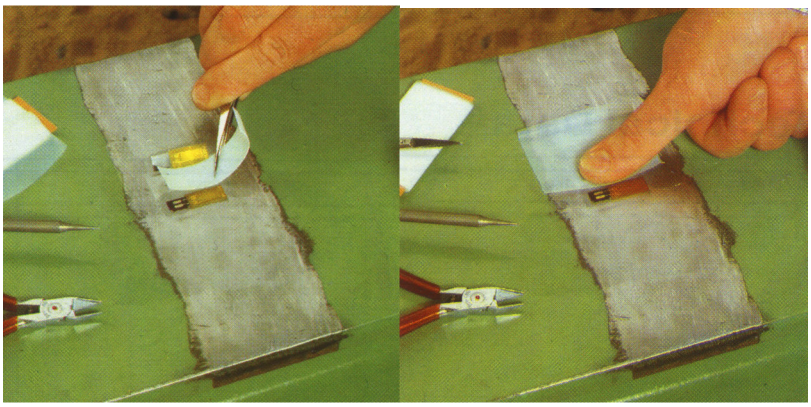
- **Gauge's preparing**

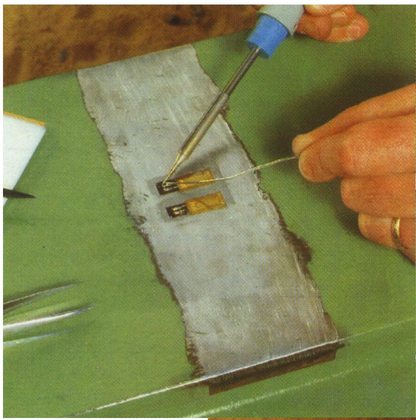
Gage's contact areas are possible to fixed by adhesive that can be also used for covering of soldering contacts. For fixing of gages we need only one operation. Gage is placed and balanced on required place and caught by adhesive on one side to make possible turning of the sensor.

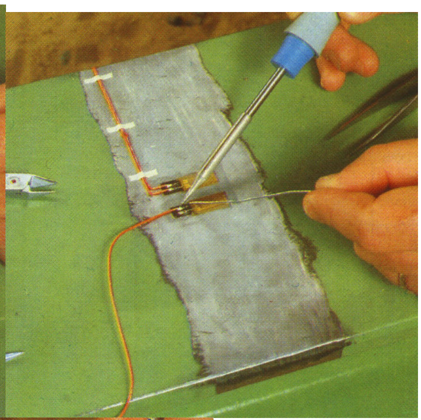


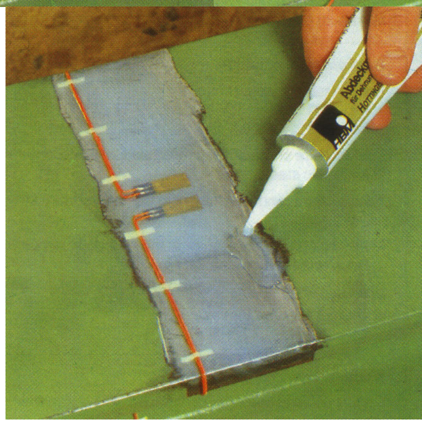
- **Procedure of gluing**

Gauge is lifted off and on glued surface is spread sufficient quantity of glue. Gage is covered up and recovered by Teflon foil. With turning of a finger can be arbitrary glue squeezed out. One has to do that very softly not to shift the grid. Under room temperature is this gage holding by finger as long as the used glue needs. If the temperature is very low, it means about zero, the gage must be held 20 or 30 minutes under the force 10 or 20N. If is possible to remove the foil without any rests we can start with soldering of wires to the grid.









- **Gluing off of strain gauge**

If there is no possibility to remove the gage by rote we can do that by acetone or by similar stuff. The small object we can put right to the solvent and on bigger objects is possible to cover grid by knot of cotton wool that is impregnate by dissolvent.

5.13.2. Applied technology of sticking

5.13.2.1. Binary fast paste X60

Size of use

This paste or glue X60 is determined for fixing of strain gages on measured surface. It is suitable for nylon's platform of the grid. Among its advantage behoove simple use and short time for gluing. X60 is proper for using on common metal, concrete, glass, porcelain and plastic. X60 consists of powdery resin **A** and liquid part **B**.

Thermal limits

Static measurement:	-200°C	till	+60°C
Dynamic measurement:	-200°C	till	+80°C

Time of setting

Time of setting depends on the temperature of environment and on temperature of glued parts. There are suitable times of setting:

Temperature [°C]	Time of setting [min] for **dynamic** meas.	Time of setting [min] for **static** meas.
20	10 – 15	20 – 30
0	50 – 60	60 – 90

5.13.2.2. Unary fast paste Z70

Z70 is unary fast paste or glue without any solvent from a branch of cyano-acrytates. Z70 is suitable for sticking of gages where the platform is made from acryl resin or nylon. Z70 connects with usual metal and plastics. It is not proper for gluing of porous material as a concrete, wood etc.

Time of setting

The speed of hardening is depended on chemical state of glued parts. Alkali materials get setting faster on the contrary acid materials can the setting slow down and in some cases also ended. The values of setting are shown in Tab. 5.1:

Material of any part	Time of setting [s]
Steel	40 – 80
Aluminum	30 – 60
Plastic	10 – 60

Tab. 5.1

The final hardness is attained after about 24 hours; the measurement is but possible after the times in Tab. 5.2:

Measurement	Temperature of gluing [°C]	
	5	20
	Shortest time of hardening [min]	
Dynamic	90	10
Static	120	15

Tab. 5.2

5.13.3. Covering

Applied strain gages need a certain protection mainly against mechanic and chemical effects. Even at ideal laboratory conditions are gage's properties changed in time if there aren't proper protections or covering. At laboratory conditions is suitable prevention from perspiration. If the conditions of environment are rougher, there is necessary to protect measuring grid as well as measured place against some fumes, fog, vapor, water, oils, warm and mechanics influence. For the first example is sufficient simple sealing or covered layer, but in the other cases must be used several covering layers from different preventing materials in order to create really good and proper preventing. An absolute protection is possible only under hermetic protection. This level of protection is often used in commercial sensors. The time of protection is given by the type of covering material, its density and also on bellicosity of medium. This time can be from some hours to a number of years.

Possible checking if the used gage is in order can be for example a value of isolation's resistance. If this value falls down from 1GΩ to 1MΩ then the zero point is changed about –60 μm/m for 120-ohm's grid, about –175 μm/m for 350-ohm's grid and about –350 μm/m for 700-ohm's grid.

The view of general protective materials

- Paint PU 100
- Paint NG 150
- Silicon's paint SL 450
- Permanent plastic cement AK 22
- Permanent plastic cement with aluminum foil ABM 75
- Transparent silicon's rubber without solvents SG 250
- Unbleached vaseline
- Micro-crystal wax
- Aluminum foil

5.13.4. Checking of installed gauges

The sensor as well as the cabling should be remitted to checking before true measuring.

Visual check

- Air bubbles under measuring grid
- Not proper glued gage, particularly on its edges
- Cold links

Rubber's check

The grid is connected to amplifier or to manual compensator fro balancing of given wiring. Then hard press rubber on grid and contacts. As a cause of that the indicator should be softly shifted. Subsequently is rubber taken away and the indicator should shift back to initial position.

Resistance of connecting cables

The resistance of cabling flows into reduction of sensitivity of measuring grid. Therefore the resistance should be measured and signed to the protocol. Known systematic error of cabling should be corrected.

Isolative resistance of measuring grid

Isolative resistance of measuring grid should be measured against the ground. Gages applied in laboratory or similar conditions should have this resistance at least 20 000 MΩ at room temperature. At out door's applications resistance should be 2 000 MΩ.

Isolative resistance of connectors

Isolative resistance of connectors depends on the quality of used isolative materials and also on their length. These resistances should attain the same values as it is for grids.

5.14. Problems of gage's connections

5.14.1. Introduction

At ordinary measurement are the strains in limits from 10^{-3} to 10^{-6} $\left[\mu m.m^{-1}\right]$. If there are used for measurement strain gages with ordinary range of the nominal resistances 120 - 600 ohms, where the k-factor is equal to 2 then the change of resistance for $R = 120\ \Omega$ is $\Delta R = \left(2{,}4.10^{-1} - 2{,}4.10^{-4}\right)\ \Omega,$. These too small values of resistance are usually measured by using Wheatston's bridge.

5.14.2. Wheatston's bridge:

A) Supply with constant voltage

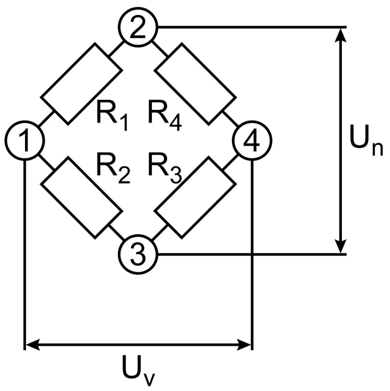


fig. 5.1

Four resistances marked as R_1, R_2, R_3 and R_4 are ordered to the bridge. Supplying diagonal between the nodal points *2* and *3* is connected to the source of constant supply voltage U_n, outgoing voltage of the bridge U_v between nodal points *1* and *4* (outgoing diagonal) is connected to the amplifier with theoretical infinite resistance.
Outgoing voltage U_v is given

$$U_v = \frac{R_1 R_3 - R_2 R_4}{\left(R_1 + R_2\right)\cdot\left(R_3 + R_4\right)} \qquad (5.1)$$

From that equation is shown that outgoing signal will be zero ($Uv = 0$), if is accepted:

$$R_1 R_3 = R_2 R_4 \qquad (5.2)$$

or

$$R_1 = R_2 = R_3 = R_4$$

If this condition is fulfilled the bridge is balanced.

Change of outgoing voltage ΔU_v is then caused by change of resistances R_l, R_2, R_3 or R_4 o ΔR_1, ΔR_2, ΔR_3 or ΔR_4. In agreement with equation (1.1) is the change of outgoing voltage in dependence on resistances' changes in the bridge given by this relation:

$$\frac{\Delta U_v}{U_n} = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right)(1-\eta) \tag{5.3}$$

Where

$$r = \frac{R_2}{R_1} = \frac{R_3}{R_4}$$

a nonlinear member η is given by

$$\eta = \frac{1}{\dfrac{1+r}{\dfrac{\Delta R_1}{R_1} + \dfrac{\Delta R_4}{R_4} + r\left(\dfrac{\Delta R_2}{R_2} + \dfrac{\Delta R_3}{R_3}\right)}} \tag{5.4}$$

If all resistances have the same value ($r = 1$), his relation will be reduced to

$$\eta = \frac{\sum_{i=1}^{4} \dfrac{\Delta R_i}{R_i}}{\left(\sum_{i=1}^{4} \dfrac{\Delta R_i}{R_i}\right) + 2} \tag{5.5}$$

From the equations (5.4) and (55) follow that nonlinearity will be for $r=1$ zero, if $\Delta R_l = -\Delta R_2$ at $\Delta R_3 = \Delta R_4 = 0$, or $\Delta R_3 = -\Delta R_4$ at $\Delta R_l = \Delta R_2 = 0$, or else $\Delta R_l = -\Delta R_2$ and at the same time $\Delta R_3 = \Delta R_4$.

This result is very important for practice and marks that bridge is behaving linearly if the same gages are connected as a active on the place of resistances R_l and R_2, or on the places R_3 and R_4 (so called half bridge connection). The nonlinearity also vanish in the case where on all four places of resistances will connect active gages with the same resistance (so called full bridge connection).

In practice is used equation (1.3) in simple form (1.6) that describes sufficiently exactly the behaving of bridge wiring in dependence on relative changes of resistance in its arms:

$$\frac{\Delta U_v}{U_n} = \frac{1}{4}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) \tag{5.6}$$

After derive of length strains we get the relation:

$$\frac{\Delta U_v}{U_n} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right) \tag{5.7}$$

Where k is k-factor of gage

Final strain is

$$\varepsilon_v = \varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4 \tag{5.8}$$

The relation (1.7) can be then written:

$$\frac{\Delta U_v}{U_n} = \frac{k}{4}\varepsilon_V \tag{5.9}$$

Also must hold true:

$$\frac{\Delta U_v}{U_n} = \frac{k_z}{4}\varepsilon_i \tag{5.10}$$

Where ε_i is indicated strain [μm/m]

Then

$$\varepsilon_i = \frac{4}{k_z} \cdot \frac{\Delta U_v}{U_n} \tag{5.11}$$

In practice k = 2, so then

$$\varepsilon_i = 2 \cdot \frac{\Delta U_v}{U_n}$$

For instance if the change of voltage is

$\frac{\Delta U_v}{U_n} = \frac{1mV}{V} = 10^{-3}$ Then $\varepsilon_i = 2 \cdot 10^{-3}\,\frac{m}{m} = 2000\,\frac{\mu m}{m}$

The strain is here in *μm/m*. Its el. image with considering (1.7) is rated unbalancing of bridge in *mV/V* (i.e. *mV* of outgoing voltage on every Volt of incoming voltage of the bridge). We can consider that the changes of two neighbors' arms will be manifested in output from bridge by difference of their values and on the contrary resistances' changes in opposite arms will be displayed in the sum. Well, we can use of these realities:

- For compensation of temperature influence (half bridge wiring),
    - For compensation of strains from inner forces that we want them to get rid of.
    - For increasing of outgoing voltage from bridge, by using of greater number of active gages.
- Removing resistive influence from incoming wires.

B) Supply with constant current

This wiring can be seen at fig. 5.15.

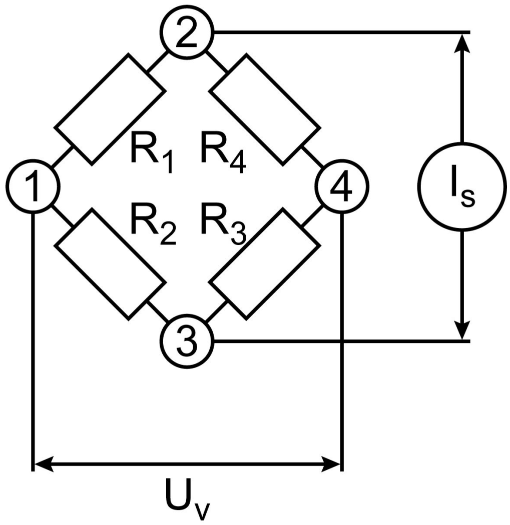


fig. 5.15

Using of constant current for feeding of circuit has the effect on increasing of sensitivity and decreases nonlinearity of Wheatston's bridge. Outgoing voltage is given:

$$U_v = \frac{i_s}{(R_1 + R_2 + R_3 + R_4)} \cdot (R_1 R_3 + R_2 R_4) \tag{5.12}$$

For balancing of bridge (*$U_v=0$*) must hold true the same equation as above, relation (1.2).
Nonlinearity of this wiring is nearly half with comparing supply with constant voltage.

5.14.3. The methods of gage's wiring to the bridge

- **Full connection** (fig. 5.16) – All arms in the bridge are placed by gages with the same resistance. The outgoing voltage will be attained peak and thermal influencing of used grids is fully compensated. It is used for the most accurate experimental measurement with highest requirements on long stability.

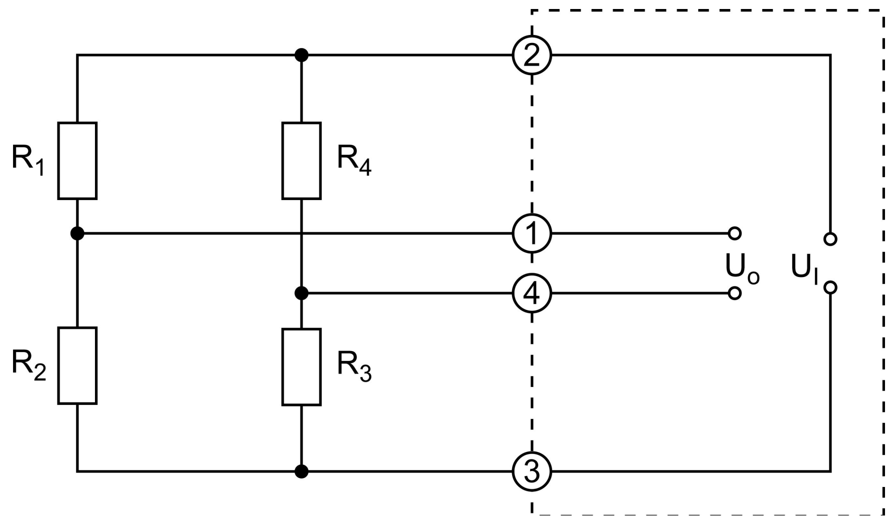


fig. 5.16

- **Half bridge connection** (fig. 5.17) – Two active gages on the place of two neighboring resistances with the mutual node on output's diagonal of the bridge (for example R_I and R_2), that are filled to the full bridge with reserve resistances. If we have the same length strains with different sign and the gages with the same resistance, is removing the nonlinearity, temperature is compensated and outgoing voltage will be half against to the full bridge. If there is no possibility to apply both gages to the half bridge as an active we use one of them as a compensating. This gage must be placed on the place with zero deformation and into the same thermal field. No compensating is standing nonlinearity owing to the measured value has this wiring behavior same as quarter bridge.

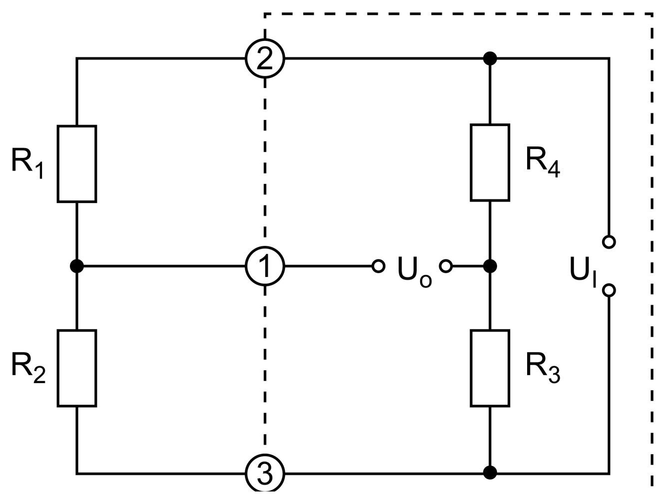


fig. 5.17

- **Quarter bridge** (fig. 5.18) – One active gauge at the place of resistance R_1, the rest is in the amplifier. The main disadvantage of this wiring is nonlinearity and thermal no stability. Thermal changes of active gauge are not compensated. At fig. 5.19 is shown three-wired connecting that can compensated thermal dependence of connection's resistance.

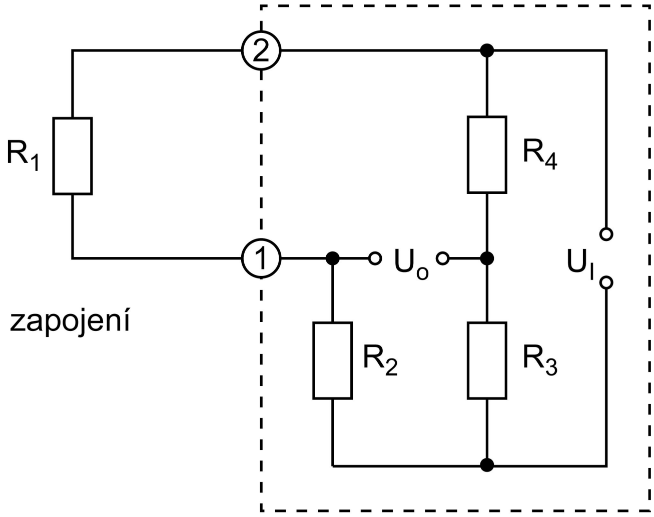


fig. 5.18

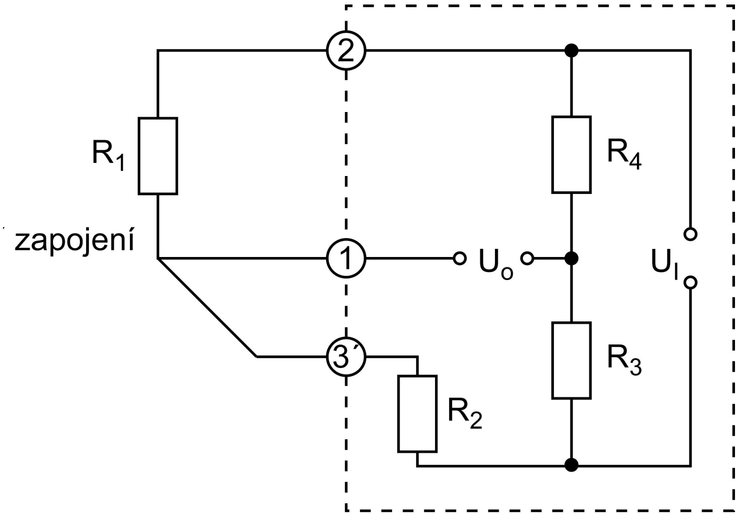


fig. 5.19

5.15. Compensation of thermal influence

One of the most important sources of errors is virtual deformation as a cause of warming of measured object. Only in the case of dynamics measuring, there is no problem with virtual deformations and their unfavorable effects.

Virtual deformations mustn’t be confused with thermal drift. It is also superposed on thermal response but it is nonreversible process.

There are several methods that can be used for removing of virtual deformation:

1. Simplest method is based on holding the state when after bridge balancing the temperature is not changing on the measured object.

2. In the second case, there is used compensating gauge, that is fixed on not strained place but the temperature field must be the same and also the material is

the same. In the quite number of practice cases can be the compensating gage used another active gage; thermal changes both of grids must be the same and the strains in the known rate and opposite signs. This is for instance at bended and tortuous bar.

3. The third method is based on using of gauges with special grid from special alloys – thermally self-compensated gages. The purpose of that compensation is to reduce the effect of thermal changes. This can be attained by chemical way of processing of measuring grid. So for example the gages made in HBM are self-compensated for ferrite steel (α $=10{,}8.10^{-6}$ /K), aluminum and its alloys (23.10^{-6} /K) and plastics (65.10^{-6} /K), austenitic steel (16.10^{-6}/K), titanium's alloys (9.10^{-6} /K), molybdenum ($5{,}4.10^{-6}$/K) and the materials on the basis of silicon ($0{,}5.10^{-6}$/K). Generally is valuable, that the closer matching between the value of self-compensated gage and thermal coefficient of tension of measured object, the lower will be apparent deformation as a cause of thermal changes after bridge balancing.

4. Finally, there is also possible use the method of numerical corrections of measured data. For that is necessary to know particular technical data of used gauge and also the temperature at the measured place.

5.15.1. Correction of apparent deformation

Correction of apparent deformation can be attained easily by using of technical parameters that are contained in every package of thermally self-compensated gauges. It is presumed, that for balancing of gage's device was at temperature T_0 and peculiar test was done at the temperature T_t. Statement on the deformation's indicator is then

$$\varepsilon_{cor}^{a} = \varepsilon_{ind} - \varepsilon_{z} \tag{5.13}$$

Where ε_{ind} is no corrected indicated value of deformation.

Apparent deformation ε_z is

$$\varepsilon_{cz} = \left[(\varepsilon_z)_{Tt} - (\varepsilon_z)_{To} \right] \tag{5.14}$$

Where $(\varepsilon_z)_{To}$, $(\varepsilon_z)_{Tt}$ are apparent deformations determined from the graph in the particular gage's package at the temperatures T_0 a T_t.

For further specification of the apparent deformation correction is achieved if we assume the reality that at the temperature of measurement is the k-factor k_T somewhat different from k-factor k at room temperature; usually is this k-factor linearly changed with temperature:

$$\frac{k_T}{k} = \frac{1+\Delta k}{k} = 1 + \frac{\Delta k}{k} = 1 + \alpha_k . \Delta T \tag{5.15}$$

In technical documents are assigned constants of sensitivity $\Delta k/k$ or thermal coefficient of sensitivity constant α_k. The value of α_k is about 100.10^{-6} /K; then for example $\Delta T = 100\,^0C$ is $k_T = 1{,}01\,k$.

The correction then is

$$\varepsilon_{cor} = \varepsilon_{cor}^{a} \cdot \frac{k_{ins}}{k_T} \tag{5.16}$$

Where from equation (4)

$$k_T = k(1 + \alpha_k . \Delta T)$$

Combination of these both corrections we attain

$$\varepsilon_{cor} = (\varepsilon_{ind} - \varepsilon_z).\frac{k_{ins}}{k_T} \tag{5.17}$$

If the gage is fixed on different materials, it means with different coefficients of thermal extension then is setting an error in self-compensation. It is assumed that for gauge on material with constant value of thermal coefficient of extension $\alpha_{S,1}$ was by the producer reserved apparent deformation $\varepsilon_{z,1}$. If the sensor is placed on material with constant value of coefficient of thermal extension $\alpha_{S,2}$ then can be determined the value of apparent deformation:

$$\varepsilon_{z,1} - \alpha_{S,1}.\Delta T = \varepsilon_{z,2} - \alpha_{S,2}.\Delta T$$

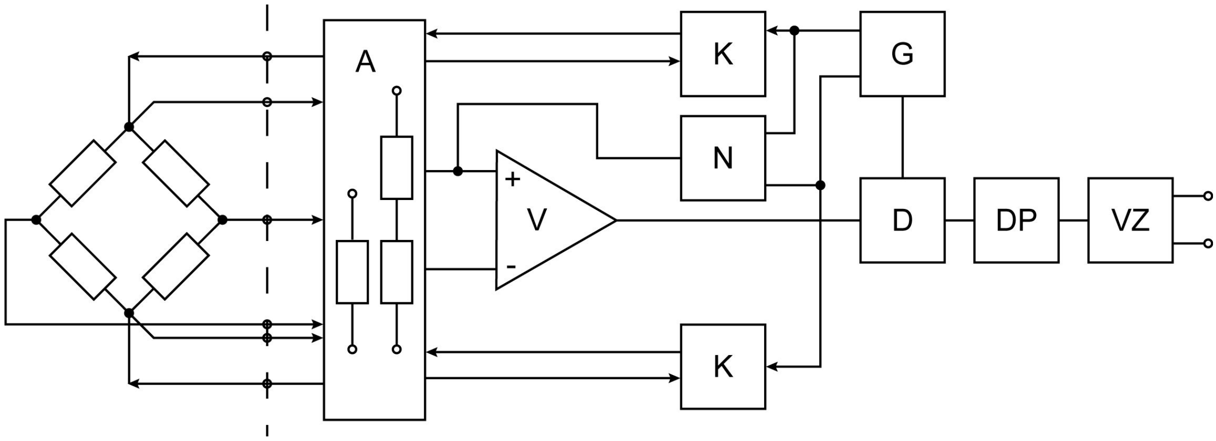
And then

$$\varepsilon_{z,2} = \varepsilon_{z,1} + \left[(\alpha_{S,2} - \alpha_{S,1}).\Delta T\right] \tag{5.18}$$

5.16. Instrumentation for measurement

5.16.1. Analog instrumentation

For demonstration of function of an experimental measuring chain can be used independent source of voltage for example voltmeter. In practice is often used special measured amplifier. The scheme of classic analog gage's amplifier for one measuring bridge is shown on following picture.



A – arbitrary circuits for complement to the full bridge
V – Incoming amplifier
K – compensation of voltage fall in cabling
G – generator of bridge supply
N – circuits for zero adjustment
D – synchronous demodulator
DP – low culvert
VZ – outgoing amplifier

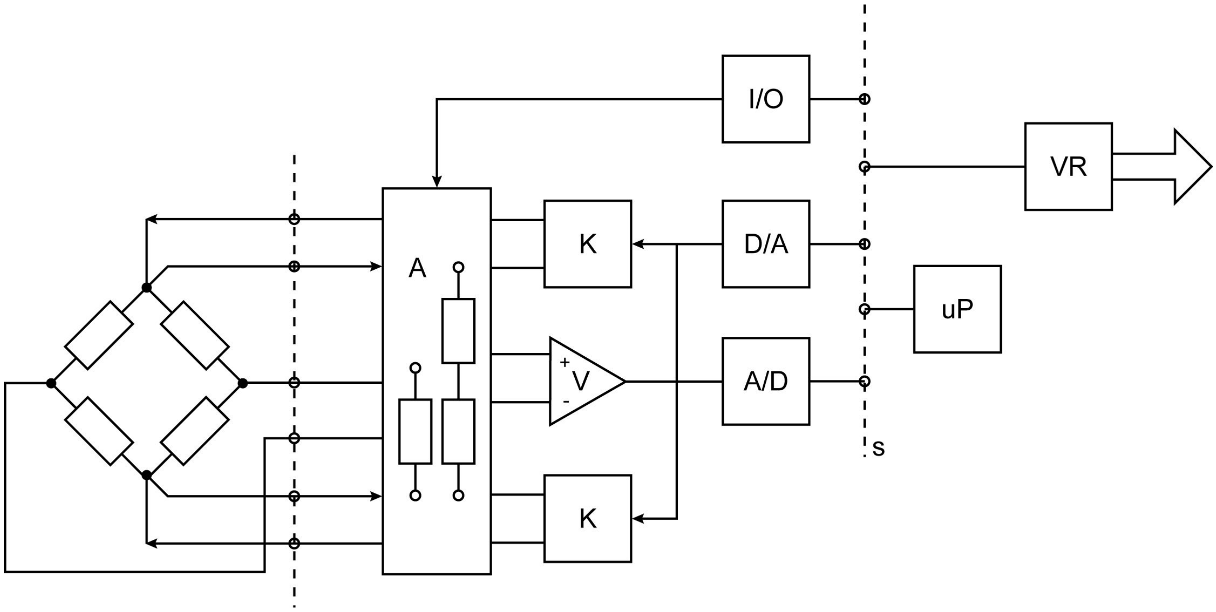
fig. 5.20

5.16.2. Digital instrumentation

5.16.2.1. Digitalization

May be the most important step in experimental analyze is digitalization of measured quantity and possibility of further working in digital shape. With increasing of accuracy and speed of A/D converters and their miniaturization was shifted analog-digital interface from output of gage's amplifier to closely behind the input amplifier. The speed of microcomputers makes possible most of basic data working in a real time.

Topical A/D converters cover thanks to the their high resolution the whole measuring rate of sensors. If there is need or required the output in the analog form so here are also in the same quality D/A converters. The scheme of digital gage's amplifier is at fig. 5.21.



A – arbitrary circuits for complement to the full bridge
V – input amplifier
K – compensation of voltage fall in cabling
D/A – outgoing converter
A/D – incoming converter measured quantity
Up – microcomputer
I/O – input / output interface
S – inner data bus of microcomputer
VR – outgoing interface

fig. 5.21

The agent of topical answer universal measuring central is for instance SPIDER8 from HBM. In one frame about size like notebook has is placed 8 inputs where each of them accepted gage's and inductive half bridge or full bridge that supply them with alternate voltage and frequency 4800Hz, further there is possibility to connect voltage +-10V, resistive sensors and current loops, thermocouple and resistive thermometers. These inputs are pre-worked by analog circuits, and then after A/D conversions are in required protocol transmit to a PC or notebook. Programmatic application is used to control the central, transmitting of the data and their working.

5.16.2.2. Digital interface

There do exist many variants how to transfer the values from gage's measuring device to PC for saving on suitable medias and for their further working. In practice there is longest time used variant inbuilt card A/D of converters that make possible to read outgoing voltage from certain number of measuring amplifier.

The higher variant is compact measuring pack containing analog pre-working signal and also true digital structure reserving distribution of digitalized data to PC for saving and further processing.

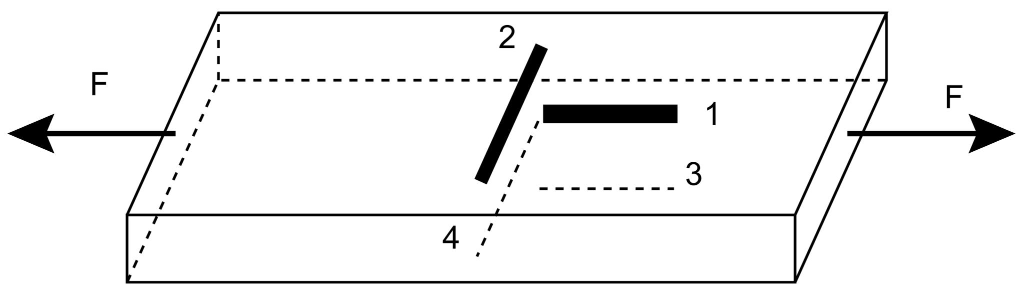
There are PCs commonly equipped by serial interfaces (COM1, COM2, ...) type RS232, and interfaces intended for printers (LPT1...) type Centronics. Both of them are given for connection of only one device. Serial interfaces COM are often used for controlling of less difficult measuring and most of numerical controlled centrals have had these interfaces. Their disadvantage is low transfer capacity (maximum units of Kbytes/s), on the contrary their advantage is simple cabling. There is possibility the interface COM modified with help simple converter to the type RS422, which makes possible through four-line-wire (RS485 through two-line-wire) of total length c. 1000m communication with the other devices equipped by this interface. Seldom are for measurement used print's interface LPT (it is used in measuring central SPIDER8 from HBM). Transfer speed of USB interface is higher though it is serial communication – it's meaning has been rising.

One of few worldwide spread interface for digital communication has become standard IEEE-488, originally connected system for measurement of US's firm Hewlett-Packard, so called HP-IB, (GP-IB), which was developed in 1972. The last modification from 1987 is called IEE488.2. That's a data bus for parallel 8 bit's data transfer with helping wires for transferring control that makes possible to connect till 16 partners with the same interface and address of partner.

In nowadays has rise the meaning of communication between computational systems through the "net" that is realized by different technical products (often is used data bus ETHERNET), some of them make possible connection to global Internet system. There is also possibility to control the application by distance control or by wireless connection. An example such a central that has among others Ethernet interface is MGCplus central of HBM Company.

5.17. Concepts of particular gages' connections

5.17.1. Tensile bar



Strain in particular direction:

$$\varepsilon_x = \frac{F}{S \cdot \mathrm{E}} = \frac{\sigma_x}{\mathrm{E}} = \frac{\sigma_N}{\mathrm{E}}$$

$$\varepsilon_y = -\mu \cdot \varepsilon_x$$

5.17.1.1. Gauge's connection

a) $\frac{1}{4}$ Bridge: connected R_1

Total strain:

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_1 = \varepsilon_{1\mathrm{M}} + \varepsilon_{1T},$$

Where ε_i - indicated strain value

$\varepsilon_{1\mathrm{M}}$ - Deformation from mechanics loading

ε_{1T} - Deformation from thermal loading

Disadvantages:

No thermal compensation – it is necessary to use self-compensated gages
No bending compensation

$$\varepsilon_1 = \varepsilon_N + \varepsilon_o = \frac{1}{E}(\sigma_N + \sigma_o)$$, Where $\varepsilon_N = \frac{\sigma_N}{E}$ and $\varepsilon_o = \frac{\sigma_o}{E}$

b) Two $\frac{1}{4}$ bridges – connected R_1, R_3

Total strain:

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_1 + \varepsilon_3$$

Where $\varepsilon_1 = \varepsilon_3 \Rightarrow \frac{2}{k} \cdot \varepsilon_i = 2\varepsilon_1 = C \cdot \varepsilon_1 \Rightarrow C = 2$ (*C* ... constant of a bridge connection)

Disadvantages:

No thermal compensation $\Rightarrow$ it is necessary to use self-compensated gages

Otherwise $\varepsilon_1 = \varepsilon_{1\mathrm{M}} + \varepsilon_{1T}$

$\varepsilon_3 = \varepsilon_{3\mathrm{M}} + \varepsilon_{3T}$

And $\varepsilon_{1M} = \varepsilon_{3M}$ a $\varepsilon_{1T} = \varepsilon_{3T}$

Then $\frac{2}{k} \cdot \varepsilon_i = 2\varepsilon_{1M} + 2\varepsilon_{1T}$

Advantages:

Bending is compensated

$$\varepsilon_{1M} = \varepsilon_N + \varepsilon_o$$
$$\varepsilon_{3M} = \varepsilon_N - \varepsilon_o$$

Then $\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_{1M} + \varepsilon_{3M} = 2\varepsilon_N$

(Providing $\varepsilon_{1T} = \varepsilon_{3T} = 0$, i.e. with using self-compensated gages. Also providing $\varepsilon_v = C \cdot \varepsilon_i$ will be $C = 2$.

c) $\frac{1}{2}$ Bridge – connected R_1, R_2

Advantages:

Thermal compensation

$$\varepsilon_1 = \varepsilon_{1M} + \varepsilon_{1T}$$
$$\varepsilon_2 = \varepsilon_{2M} + \varepsilon_{2T}$$

And $\varepsilon_{1T} = \varepsilon_{2T}$

Then $\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_{1M} - \varepsilon_{2M}$

Higher sensitivity

$\varepsilon_{2M} = -\mu\varepsilon_{1M} = -0{,}3\varepsilon_{1M}$,

i.e. $\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = 1{,}3 \cdot \varepsilon_{1M}$, $\Rightarrow \quad C = 1{,}3$

Disadvantages:

No bending compensation

$$\varepsilon_{1M} = \varepsilon_N + \varepsilon_o = \frac{1}{E}(\sigma_N + \sigma_o) = \frac{2}{k} \cdot \frac{\varepsilon_i}{1{,}3}$$

From where $\sigma_N + \sigma_o = \frac{2}{k} \cdot \frac{\varepsilon_i}{1{,}3} \cdot E$

d) Full bridge - R_1, R_2, R_3, R_4 all gauges connected

Advantages:

Thermal compensation

$$\varepsilon_{1T} = \varepsilon_{2T} = \varepsilon_{3T} = \varepsilon_{4T}$$

Then $\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_{1M} - \varepsilon_{2M} + \varepsilon_{3M} - \varepsilon_{4M}$

Higher sensitivity

$$\varepsilon_{2M} = -\mu\varepsilon_{1M}$$
$$\varepsilon_{4M} = -\mu\varepsilon_{3M}$$

Then $\varepsilon_V = (1+\mu) \cdot (\varepsilon_{1M} + \varepsilon_{3M})$

And for simple tension must be true: $\varepsilon_{1M} = \varepsilon_{3M}$

$$\varepsilon_V = 2(1+\mu) \cdot \varepsilon_{1M} = 2{,}6\varepsilon_{1M} \quad \Rightarrow \quad C = 2{,}6$$

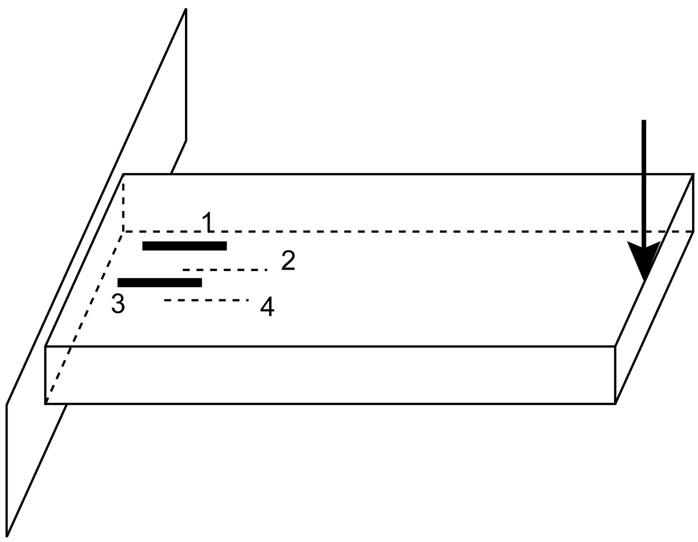
Bending compensation

$$\varepsilon_{1M} = \varepsilon_N + \varepsilon_o$$
$$\varepsilon_{3M} = \varepsilon_N - \varepsilon_o$$

Then $\frac{2}{k}\varepsilon_i = \varepsilon_V = 2{,}6 \cdot \varepsilon_N = 2{,}6\frac{\sigma_N}{\mathrm{E}}$

From where $\sigma_N = \frac{2}{k} \cdot \frac{\varepsilon_i}{2{,}6} \cdot \mathrm{E}$

5.17.2. Bended bar



Extreme bending stress:

$$\sigma_o = \frac{M_o}{W_o} = E \cdot \varepsilon_o$$

5.17.2.1. Gauge's connection

a) $\frac{1}{4}$ **Bridge – connected** R_1 **or** R_2

Total strain:

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_1 = \varepsilon_{1M} + \varepsilon_{1T},$$

$$\varepsilon_{1M} = \varepsilon_N + \varepsilon_o = \frac{1}{E}(\sigma_N + \sigma_o)$$

$$\sigma_N + \sigma_o = \frac{2}{k} \cdot \varepsilon_i \cdot E$$

Where ε_i - indicated strain value

ε_{1M} - Deformation from mechanics loading

ε_{1T} - Deformation from thermal loading

Disadvantages:

No thermal compensation **-** it is necessary to use self-compensated gages
No bending compensation

b) $\frac{1}{2}$ **Bridge – connected** R_1**,** R_2

Total strain:

$$\varepsilon_V = \frac{2}{k}\varepsilon_i = \varepsilon_{1M} - \varepsilon_{2M}$$

Advantages:

Thermal compensation

$$\varepsilon_{1T} = \varepsilon_{2T}$$

Higher sensitivity during bending

$$\varepsilon_{1M} = -\varepsilon_{2M}$$

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = 2 \cdot \varepsilon_{1\mathrm{M}} \quad \Rightarrow \quad C = 2$$

Tension compensation

$$\varepsilon_{1\mathrm{M}} = \varepsilon_N + \varepsilon_o$$

$$\varepsilon_{2\mathrm{M}} = \varepsilon_N - \varepsilon_o$$

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = 2 \cdot \varepsilon_o = 2\frac{\sigma_o}{\mathrm{E}}$$, i.e. $$\sigma_o = \frac{1}{k} \cdot \varepsilon_i \cdot \mathrm{E}$$

c) Full bridge - R_1, R_2, R_3, R_4 all gauges are connected

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_{1\mathrm{M}} - \varepsilon_{2\mathrm{M}} + \varepsilon_{3\mathrm{M}} - \varepsilon_{4\mathrm{M}}$$

Advantages:

Thermal compensation

$$\varepsilon_{1T} = \varepsilon_{2T} = \varepsilon_{3T} = \varepsilon_{4T}$$

Higher sensitivity

$$\varepsilon_{1\mathrm{M}} = \varepsilon_{3\mathrm{M}}$$

$$\varepsilon_{2\mathrm{M}} = \varepsilon_{4\mathrm{M}} = -\varepsilon_{1\mathrm{M}}$$

$$\varepsilon_V = 4 \cdot \varepsilon_{1\mathrm{M}} \quad \Rightarrow \quad C = 4$$

Tension compensation

$$\varepsilon_{1\mathrm{M}} = \varepsilon_{3\mathrm{M}} = \varepsilon_N + \varepsilon_o$$

$$\varepsilon_{2\mathrm{M}} = \varepsilon_{4\mathrm{M}} = \varepsilon_N - \varepsilon_o$$

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = 4 \cdot \varepsilon_o = 4\frac{\sigma_o}{\mathrm{E}}$$, i.e. $$\sigma_o = \frac{1}{2k} \cdot \varepsilon_i \cdot \mathrm{E}$$

5.17.3. Torsion

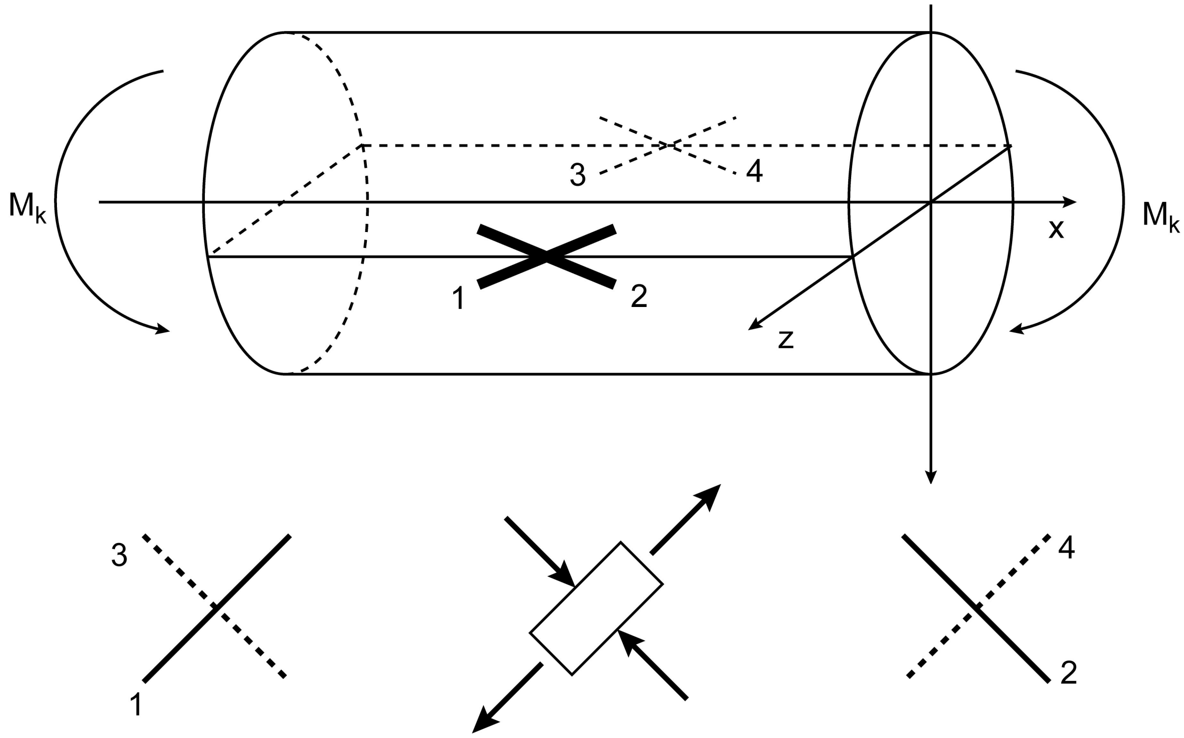


fig. 5.22

Stress and strain at a point from twisting moment M_k:

$$\gamma = \frac{\tau_{\max}}{G}$$

Where

$$\varepsilon_{45} = \frac{\gamma}{2}$$

$$\tau_{\max} = \frac{M_K}{W_K}$$

$$\tau_{\max} = G \cdot \gamma = G \cdot 2\varepsilon_{45}$$

Let us say $M_K = \tau_{\max} \cdot W_K$

For circle cross section area:

$$W_K = \frac{\pi}{16} d^3 \approx \frac{1}{5} d^3$$

5.17.3.1. Gauge's connection

a) $\frac{1}{2}$ **Bridge – connected** R_1, R_2

Total strain:

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_1 - \varepsilon_2$$

Where $\varepsilon_1 = \varepsilon_{1\mathrm{M}} + \varepsilon_{1T}$

$$\varepsilon_2 = \varepsilon_{2\mathrm{M}} + \varepsilon_{2T}$$

$$\varepsilon_{1\mathrm{M}} = -\varepsilon_{2\mathrm{M}} = \varepsilon_{45}$$

Advantages:

Thermal compensation

$$\varepsilon_{1T} = \varepsilon_{2T}$$

Then $\varepsilon_v = \frac{2}{k} \cdot \varepsilon_i = 2\varepsilon_{45} \quad \Rightarrow \quad C = 2$

Compensation of axial force

$$\varepsilon_{1N} = \varepsilon_{2N}$$

b) Full bridge - R_1, R_2, R_3, R_4 all gauges are connected

Advantages:

The same adv. as a half bridge

Higher sensitivity

$$\varepsilon_v = \frac{2}{k} \varepsilon_i = 4\varepsilon_{45} \qquad \Rightarrow \quad C = 4$$

Where $\varepsilon_{45} = \frac{1}{2k} \cdot \varepsilon_i$

$$\tau_{\max} = G \cdot 2\varepsilon_{45} = G \cdot \frac{1}{k} \cdot \varepsilon_i$$

Strain compensation of a bending moment M_Z, $(\varepsilon_{1o})_Z = (\varepsilon_{2o})_Z$

Strain compensation of a bending moment M_y, $(\varepsilon_{1o})_y = (\varepsilon_{2o})_y$

5.17.4. Shear stress

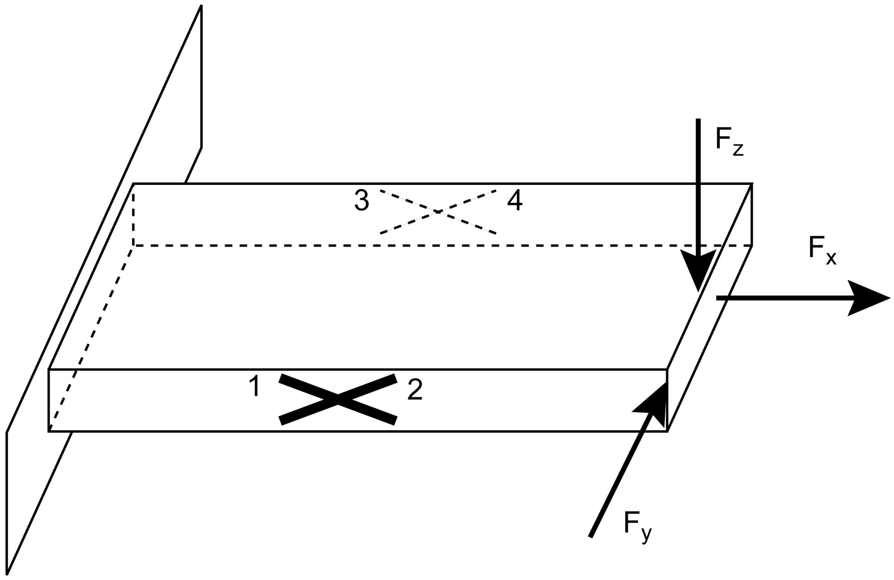


fig. 5.23

Stress and strain at a point of neutral axis from force F_Z:

$$\tau = \gamma \cdot G = 2\varepsilon_{45} \cdot G$$

Where

$$\varepsilon_{45} = \frac{\gamma}{2}$$

For rectangle cross-section:

$$\tau_{\max} = \frac{3}{2} \cdot \frac{T}{S}$$

5.17.4.1. Gauge's connection

a) $\frac{1}{2}$ **Bridge – connected** R_1, R_2

Total strain:

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_1 - \varepsilon_2$$

Advantages:

Thermal compensation

$$\varepsilon_{1T} = \varepsilon_{2T}$$

$$\varepsilon_V = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_{1\mathrm{M}} - \varepsilon_{2\mathrm{M}}$$

And $\varepsilon_{1M} = -\varepsilon_{2M} = \frac{\gamma}{2} = \frac{\tau_{max}}{2G}$, where $\tau_{max} = \frac{3}{2} \cdot \frac{F_Z}{S}$

Then $\varepsilon_v = \frac{2}{k} \cdot \varepsilon_i = 2\varepsilon_{1M} = \gamma = \frac{\tau_{max}}{G}$

$$\tau_{max} = \frac{2}{k} \cdot \varepsilon_i \cdot G$$

Compensation of bending moment from force F_Z

Compensation of shifting force T and bending moment from force F_y

$\tau = 0$, $\gamma = 0 = \varepsilon_{45} = \varepsilon_{135}$

$\varepsilon_{1M} = \varepsilon_{2M}$

Compensation of force F_x

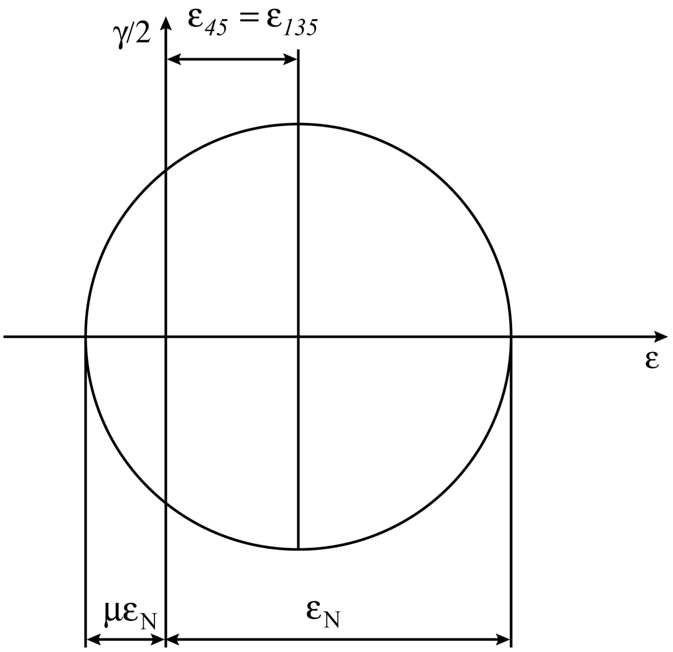


fig. 5.24

b) Full bridge - R_1, R_2, R_3, R_4 all gauges are connected

Total strain:

$$\varepsilon_{1M} = \varepsilon_{3M} = -\varepsilon_{2M} = -\varepsilon_{45}$$

And $\varepsilon_{1M} = \frac{\gamma}{2} = \frac{\tau}{2G}$

Then $\varepsilon_v = \frac{2}{k} \cdot \varepsilon_i = \varepsilon_{1M} - \varepsilon_{2M} + \varepsilon_{3M} - \varepsilon_{4M}$

Let as say $\varepsilon_v = 4\varepsilon_{1M} \Rightarrow C = 4$

$$\varepsilon_v = \frac{2}{k} \cdot \varepsilon_i = 4 \cdot \frac{\tau}{2G}$$

$$\tau = \frac{1}{k} \varepsilon_i \cdot G$$

Advantages:

Thermal compensation

$$\varepsilon_{1T} = \varepsilon_{2T} = \varepsilon_{3T} = \varepsilon_{4T}$$

Compensation of bending moment from force F_Z

Compensation of bending moment and shear from F_y

Compensation of a normal force F_x

5.17.5. Gauge measuring main stresses

These relations are true if we know main directions.

From Mohr's circle:

$$\varepsilon_\alpha = \frac{\varepsilon_1 + \varepsilon_2}{2} + \frac{\varepsilon_1 - \varepsilon_2}{2} \cdot \cos 2\alpha$$

If is presumed

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1$$

The deformation at direction is

$$\varepsilon_\alpha = \frac{\varepsilon_1}{2}(1 + \cos 2\alpha) + \frac{\varepsilon_2}{2}(1 - \cos 2\alpha) = \varepsilon_1 \cos^2 \alpha + \varepsilon_2 \cdot \sin^2 \alpha$$

Or

$$\varepsilon_\alpha = \cos^2 \alpha \left(\varepsilon_1 + \varepsilon_2 \cdot tg^2 \alpha\right)$$

If

$$tg^2 \alpha = \mu \text{ , i.e. } \alpha = arctg\sqrt{\mu}$$

and with the true relation

$$\cos\alpha = \frac{1}{\sqrt{1 + tg^2\alpha}} \text{ , Or } \cos^2 \alpha = \frac{1}{1 + tg^2 \alpha} = \frac{1}{1 + \mu}$$

Is the strain in the direction

$$\varepsilon_\alpha = \cos^2\alpha(\varepsilon_1 + \mu\varepsilon_2)$$

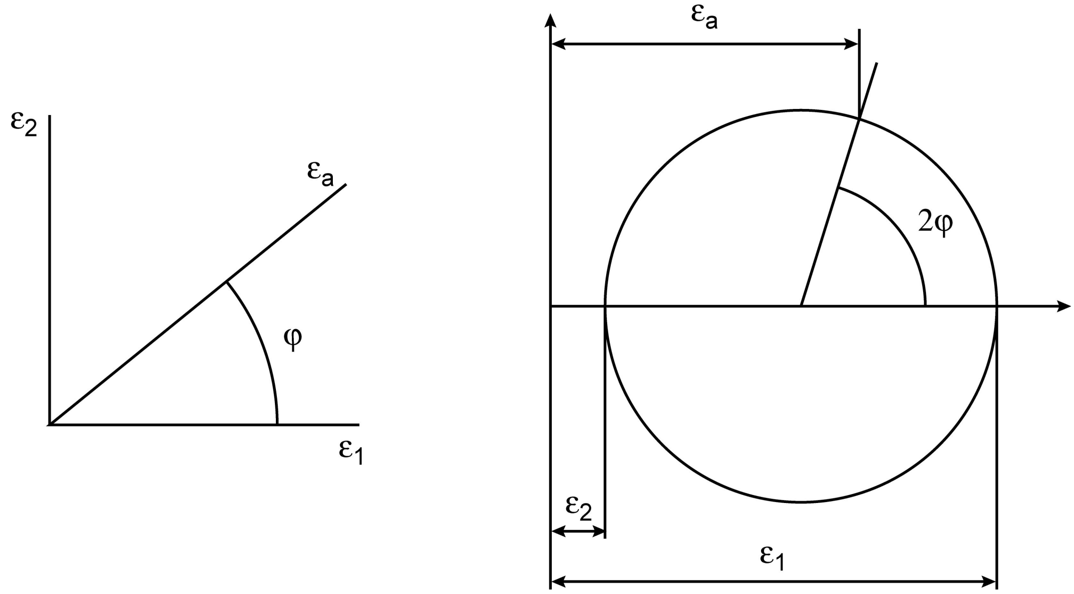


fig. 5.25

Main stress:

$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2)$$

From where

$$(\varepsilon_1 + \mu\varepsilon_2) = \frac{\sigma_1(1-\mu^2)}{\mathrm{E}} \qquad \Rightarrow \qquad \frac{\varepsilon_\alpha}{\cos^2\alpha} = \frac{\sigma_1(1-\mu^2)}{\mathrm{E}}$$

i.e.

$$\varepsilon_\alpha(1+\mu) = \frac{\sigma_1}{\mathrm{E}}(1+\mu)(1-\mu)$$

$$\sigma_1 = \frac{\mathrm{E}}{1-\mu} \cdot \varepsilon_\alpha$$

5.17.6. Plain stress with unknown directions of main stresses

Following relations are valid for gauge's rosettes of the type - $0^0/45^0/90^0$.

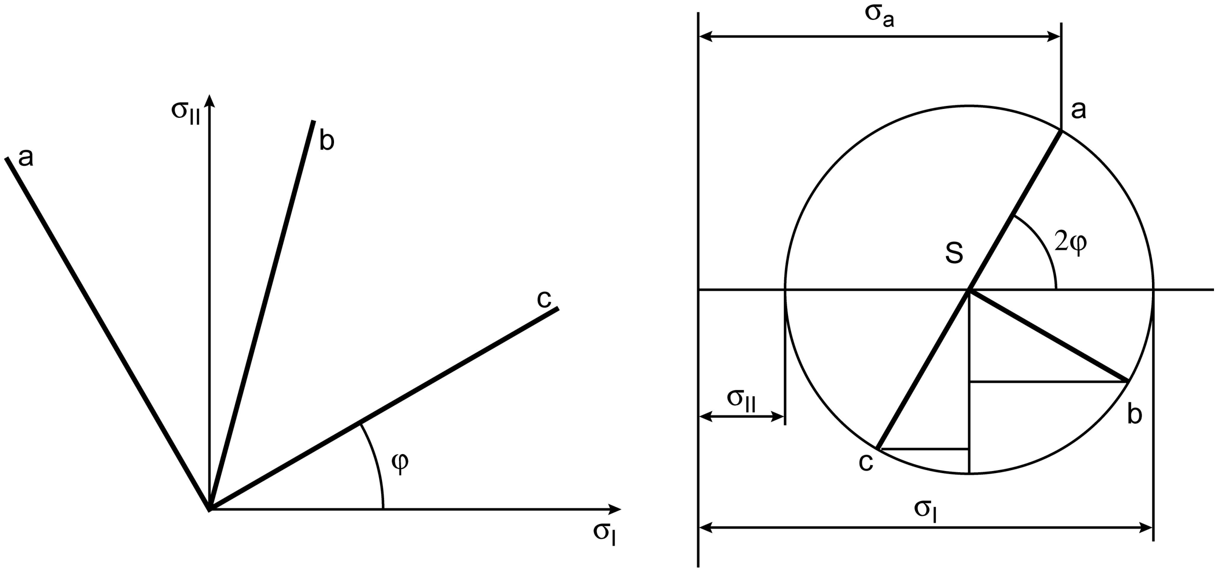


fig. 5.26

Position of the centre from Mohr's circle:

$$A = \frac{\varepsilon_a + \varepsilon_c}{2}$$

Radius of Mohr's circle:

Nominator	Denominator	$2\alpha_a$
+	+	0° – 90°
+	-	90° – 180°
-	-	180° – 270°
-	+	270° - 360°

$$B = \sqrt{(\varepsilon_a - A)^2 + (\varepsilon_b - A)^2}$$

Main strain:

$$\varepsilon_{I,II} = A \pm B$$

The angle of main axis:

$$tg2\alpha_a = \frac{\varepsilon_b - A}{\varepsilon_a - A}$$

Main stresses:

$$\sigma_I = \frac{E}{1-\mu^2}(\varepsilon_I + \varepsilon_{II}) = \frac{E}{1-\mu^2}[A + B + \mu A - \mu B]$$

tj. $$\sigma_{I,II} = \frac{E}{1-\mu} \cdot A \pm \frac{E}{1+\mu} \cdot B$$

5.17.7. Plain stress with known directions of main stresses

Main stresses**:**

$$\sigma_1 = \frac{E}{1-\mu^2}\left(\varepsilon_1 + \mu\varepsilon_2\right)$$

$$\sigma_2 = \frac{E}{1-\mu^2}\left(\varepsilon_2 + \mu\varepsilon_1\right)$$

MEASURING OF SHEAR STRESS WITH COMPENSATION OF A NORMAL STRESS

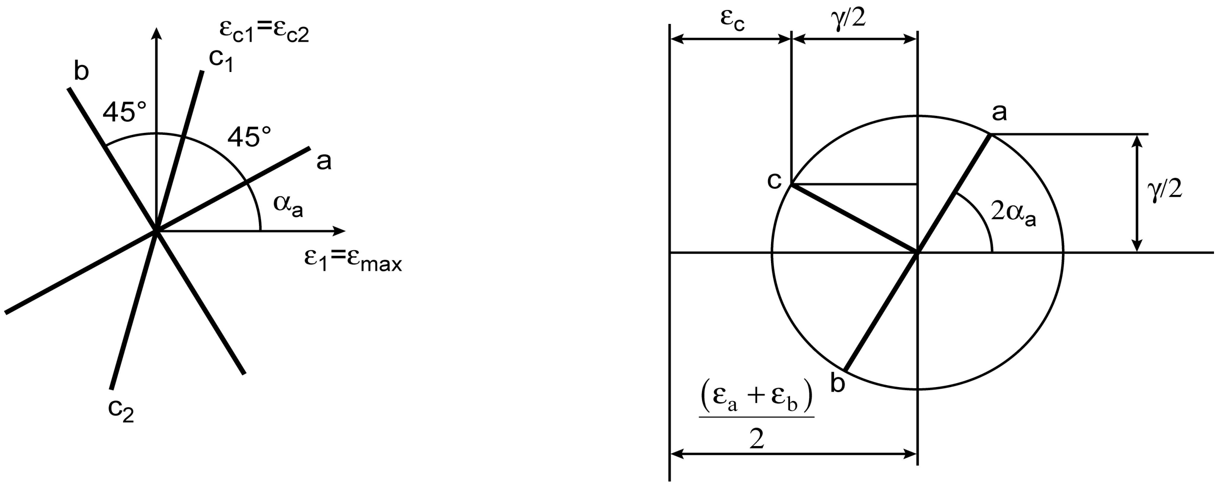


fig. 5.27

From Mohr's circle is possible to determine:

$$\frac{\gamma_a}{2} = \frac{\varepsilon_a + \varepsilon_b}{2} - \varepsilon_c$$

Or

$$\gamma_a = \varepsilon_a + \varepsilon_b - 2\varepsilon_c$$

Shear stress in the direction '*a*'

$$\tau_a = G \cdot \gamma_a = \frac{E}{2(1+\mu)} \cdot \cdot \gamma_a$$

Or

$$\tau_a = \frac{E}{2(1+\mu)} \cdot (\varepsilon_a + \varepsilon_b - 2\varepsilon_c)$$

5.18. Literature

[1] Hoffmann, K.: An Introduction to Measurements using Strain Gages, Hottinger Baldwin Messtechnik GmbH, Darmstad